

ESSAYS IN FINANCIAL ECONOMICS AND RISK MANAGEMENT

A Dissertation

by

LIN ZOU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2007

Major Subject: Economics

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ABSTRACT

Essays in Financial Economics and Risk Management. (August 2007)

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Dr. Qi Li

This dissertation contains three essays. Chapter II compares the performance of different Value-at-Risk (VaR) models during the Asian financial crisis. Since VaR is widely used as a measure of risk, various methodologies have been suggested for estimating VaR. Authors including Tsay (2002) point out that different estimators can give very different results for VaR. Bao et al. (2006) look at predictive accuracy of seven different estimators during the Asian crisis using coverage probabilities. We take a different approach to evaluating VaR estimators and look at their performance when used as a portfolio selection tool. In particular, we look at an investor with a downside risk constraint and ask how VaR estimators perform in terms of portfolio performance over the Asian crisis period. Our findings indicate that the VaR estimator with the best coverage probability is not necessarily the best estimator in terms of portfolio performance.

Chapter III investigates the dynamic relation between stock returns and volume of individual stocks. We model the market makers who accept bid-ask offers, so the instantaneous demand may not equal supply at each transaction price. A mini-Exchange platform has been developed by Su (2007) to simulate the trading process. And the simulation results suggest that during the price adjustment periods relatively low trading volume predicts a large absolute value in price change in

the future. We implement a mixture normal approach to estimate the relationship between daily return and past trading volume for individual stocks. The empirical results are consistent with the model prediction.

Chapter IV looks at international diversification of equity portfolios and currency hedging during the Asian Crisis. We take the view of a safety-first perspective, and show how Roy's safety first criterion can be used to guide portfolio selection and hedging. We demonstrate how the safety first criterion can be implemented by exploiting the fat tail property of asset returns and the statistical theory of extremes. We document how such portfolios would perform during the Asian Crisis, a stern test for a downside-risk constraint, as the Asian Crisis was marked by significant currency devaluations and negative local currency returns on equity portfolios.

To My Parents

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CHAPTER I

INTRODUCTION

Chapter II investigates the performance of seven different Value-at-Risk (VaR) models. VaR is a standard measure of market risk. It is used to ensure that financial institutions can still be in business after a catastrophic event. Many methodologies have been developed to calculate VaR. But due to the uncertainty distribution of returns, the estimates of VaR can differ substantially across models. Therefore, an essential problem is to determine which model gives a more accurate estimate. Unfortunately, the comparison is not easy to carry out since the true VaR is unknown, even after the fact. Alternatively, in this chapter I am asking a more relevant question which VaR estimation method yields the highest returns.

My major goal in Chapter II is to analyze the performance of different VaR models in terms of portfolio performance. There are many applications of VaR. One of them is to guide the portfolio selection of financial practitioners. In order to use VaR in portfolio construction, we first look at the investors with asymmetric preferences towards portfolio returns. Those investors worry more about the downside risk and want to limit the risk of having a significant negative return on a portfolio. In such case, it is reasonable to consider investors' portfolio optimization as subject to a downside risk constraint, i.e., a VaR constraint. With a VaR constraint, the investors' problem now can be formalized by a lexicographic safety-first rule. According to the safety-first rule, once investors secure that the chance of having a large negative return is lower than a certain probability, the investors will maximize their expected returns.

I then demonstrate safety first portfolio selection guiding by seven VaR mod-

The journal model is Journal of Econometrics.

els. Those models are RiskMetrics, Historical Simulation (HS), Monte-Carlo Simulation (MC), Nonparametric Estimation (NP), Generalized Extreme Value Distribution (GEV), Generalized Pareto Distribution (GPD) and Hill Estimator (Hill). The last three models belong to the family of extreme value theory. The fundamental difference is their assumptions on the return distribution. In empirical analysis, the hypothetical investor has the choice of two international stock index funds. Based on the estimates of VaR, he chooses the optimal portfolio composition and the optimal amount of borrowing. I found the main disparity across methods is in the optimal amount of borrowing. I also found GPD and Hill methods are more conservative methods, while MC and RiskMetrics methods are more aggressive methods. During the sample period, MC method out performances other methods. The empirical findings also indicate that the VaR estimator with the best coverage probability is not necessarily the best estimator in terms of portfolio performance.

The focus of Chapter III is how past volume affects future prices. The theoretical model we built up in this chapter captures many features in real world. If there is an investor who wants to buy some shares of a stock, she can either buy at current market price, or she can submit a limit order with how many shares she wants to buy and at which price she wants to buy. The information, which is available to her before she makes her investment decision, includes public information about the stock and past transaction information, e.g., the last transaction price and bid-ask spread on the stock. She may also receive some private information on this stock. This is the exact kind of investors we model in this chapter, except we only consider limit orders at the current stage.

Therefore, in every trading period, each trader allocates her wealth between cash and stock to maximize her utility function. The fundamental value of the stock is unknown to the traders, but the distribution of the fundamental value is public

information. Each trader also receives her private signal on the fundamental value, which equals to fundamental value plus an error term. Based on both common and private information, traders will generate their optimal limit orders. The model has been solved by Su (2007). He developed an exchange platform called MiniExchange to match the buy and sell orders. While he simulates the trading process, he records trading volume and price for every trading period. His analytical findings show that a relatively low past trading volume indicates a larger price movement in the future.

In order to check our model, we test the model prediction with real data. Our empirical estimation confirms model prediction. In our empirical estimation, in order to capture the relation between return and volume under different information flows, we apply mixture normal distribution to stock returns. The estimation results are consistent with the model predictions.

Chapter IV looks both equity portfolio selection and currency hedging during the Asian Crisis. It's more like an extension of Chapter II with currency hedging included. We again use Roy's safety first criterion to guide portfolio selection, and further look at hedging decisions from the unique perspective of an investor with a VaR constraint (downside risk constraint). So VaR estimation is also involved in this chapter. We use the extreme value theory to estimate the tail distribution of returns. The hypothetical U.S. investor in this chapter invests in a portfolio consisting of a U.S. equity index and an equity index from one of three Asian economies, Indonesia, Korea, and Thailand. The sample period we choose includes the Asian financial crisis of 1997. We find that safety-first portfolios performed better than buy-and-hold portfolios of U.S. or foreign stocks. During the actual Asian Crisis period the safety-first portfolios outperformed the alternatives. We also find that hedging and portfolio proportions are important to improve portfolio performance.

CHAPTER II

VALUE-AT-RISK MODELS IN PORTFOLIO SELECTION

A. Introduction

Value at Risk (VaR) has been widely used as the standard measure of market risk. It is defined as the maximal value that a portfolio will lose for a given probability during a given period. VaR has many applications: financial institutions use it to assess their risk and regulators use it to set margin requirements. Consequently, having accurate estimates of VaR is important.

Many methodologies have been developed to calculate VaR. Among these different approaches, the estimates of VaR can differ substantially due to uncertainty regarding the distribution of returns. Tsay (2002) gives empirical examples of computing VaR by different approaches and shows that the estimates of VaR vary greatly across different methods. Some of the difference can be 0.1% of the financial position, which can be quite significant for a big turnover. Therefore, an essential problem is to determine which model gives the more accurate estimate. Unfortunately, the comparison is not easy to carry out since the true VaR is unknown. Tsay (2002) suggests that one can gain insight into the range of VaR by applying several methods. However, the estimates of VaR from different methods can vary a lot, eg. the range of estimates might vary from -1% to -7% of portfolio wealth. Even if one applies all methods, there may still be no conclusion regarding which method is the best.

Several recent papers have worked on a comparison of models. Manganelli and Engle (2001) provide a comprehensive survey of the recent developments in VaR modelling, and they evaluate the performance of these methods using a Monte Carlo simulation. Their results show that conditional autoregressive Value at Risk (CAViaR)

models are the best performers when there is a heavy-tailed Data Generating Process (DGP). However, since the real DGP is unknown, their results are not definitive.

Bao, Lee and Saltoglu (2006) investigate the performance of different VaR models by using the stock market data of five Asian economies who suffered from the 1997-1998 financial crisis. They use two criteria to compare the predictive accuracy of VaR estimates. The Asian Crisis provides a type of stress test for VaR estimators. However, despite a careful and detailed study, their results are inconclusive, as they cannot say which model performs best.

Bao et al (2006) investigate performance on statistical grounds - which VaR estimation method is most accurate. Alternatively, we might ask which method of estimating VaR yields the highest return. This is in some ways the most relevant question, since there may be periods such as crises or turning points that 'count more' in portfolio performance. A method that performs well in these periods may be preferred even if it is less accurate overall. A complication, however, is that this last question requires us to take a stand on how VaR is used in portfolio construction.

One of the uses of VaR is to guide the portfolio selection of financial practitioners. According to a survey by the Stern School of Business at New York University, 60% of pension funds that responded to a survey said they make use of VaR. For investors with asymmetric preferences towards portfolio returns, downside risk may be of special concern. In this case, it is reasonable to consider investors' portfolio optimization as subject to a downside risk constraint, i.e., a VaR constraint. A lexicographic safety-first rule can be used to formalize an investors' problem with a VaR constraint. Since VaR is incorporated in safety-first portfolio selection, we can then compare different methods of estimating VaR in terms of portfolio performance.

The remainder of the paper is organized as follows. Section 2 describes the safety-first portfolio problem and alternative estimators of VaR. Section 3 defines data

and presents the exact procedures we followed to carry out the portfolio selection of safety-first investors. Section 4 demonstrates how choice of VaR models can differ the combination of portfolio and its return and provides some empirical results. Section 5 contains our conclusion.

B. Safety First Rule

1. Safety First Portfolio Selection

A generalized lexicographic form of the safety-first principles was introduced by Roy (1952) and Telser (1955). Roy's criterion was extended by Arzac and Bawa (1977) to allow borrowing and lending. Arzac and Bawa consider a single-period portfolio problem, although, we can extend the problem to multiple periods. At time t , given initial wealth W_t , an investor makes his optimal choice of both risk-free and risky assets to maximize his next period's wealth. Let b_t denotes the optimal amount of borrowing¹ at time t , X_{jt} is the optimal amount of risky asset j that the investor buys at time t , and P_{jt} is the price of the risky asset j at t . A safety-first investor faces the budget constraint: $\sum_j P_{jt}X_{jt} - b_t = W_t$. At period $t + 1$, the investor will receive $\sum_j P_{j,t+1}X_{jt}$ from the risky asset investment. After paying back the amount he borrowed in period t , he ends up with $V_{t+1} = \sum_j P_{j,t+1}X_{jt} - b_t(1 + r_t)$, where V_{t+1} is investor's wealth at $t + 1$ and r_t is the interest rate for loans contracted at t and maturing at $t + 1$. Other than maximizing the expected wealth V_{t+1} , a safety-first investor is also concerned about downside portfolio risk. We formalize this as follows: The probability Pr is the chance that the next period's return is smaller than a given critical value s , where s is considered a disaster level of wealth. We set $\pi = 1$ if Pr is less than the investor's maximal acceptable probability (α) of this disaster loss value

¹ $b > 0$ means borrowing; $b < 0$ means lending.

s , i.e. $Pr < \alpha$. Otherwise, $\pi = 1 - Pr$. The investor then maximizes the expected return V_{t+1} for a given π at time t .

The portfolio selection problem can be summarized as:

$$Max_{X_{jt}, b_t}(\pi, E_t(V_{t+1})) \quad \text{subject to} \quad \sum_j P_{jt} X_{jt} - b_t = W_t$$

$$\text{where} \quad \pi = 1 \text{ if } Pr = Pr\left(\sum_j P_{jt+1} X_{jt} - b_t(1 + r_t) \leq s\right) \leq \alpha$$

$$\pi = 1 - Pr \text{ otherwise}$$

$$\text{and } V_{t+1} = \sum_j P_{jt+1} X_{jt} - b_t(1 + r_t)$$

The net return of the risky asset portfolio is

$$R_{t+1} = \left(\sum_j P_{jt+1} X_{jt}\right) / \left(\sum_j P_{jt} X_{jt}\right) - 1. \quad (2.1)$$

By definition we know $R_{t+1} = \sum_j \gamma_{jt} R_{jt+1}$, where γ_{jt} is the fraction of risky asset j bought by investor at time t and R_{jt+1} is risky asset j 's return. With equation (1) and the budget constraint $\sum_j P_{jt} X_{jt} = W_t + b_t$, the final value of the portfolio at $t+1$ can be written as

$$\sum_j P_{jt+1} X_{jt} - b_t(1 + r_t) = (W_t + b_t)(1 + R_{t+1}) - b_t(1 + r_t) = W_t(1 + r_t) + (W_t + b_t)(R_{t+1} - r_t) \quad (2.2)$$

Arzac and Bawa (1977) proved that a Safety-first investor will always buy some risky assets and the amount bought will satisfy

$$W_t + b_t = \frac{s - W_t(1 + r_t)}{q_\alpha(R_{t+1}) - r_t}, \quad (2.3)$$

where $q_\alpha(R_{t+1})$ is the α -th quantile which can be estimated by any of the VaR models.

Therefore, the optimization problem can be reduced to

$$\max_{\gamma_{jt}, b_t}(E_t(V_{t+1})) = W_t(1 + r_t) - [s - W_t(1 + r_t)] \frac{E_t(R_{t+1}) - r_t}{(r_t) - q_\alpha(R_{t+1})}$$

The portfolio selection problem now can be separated into two stages. First, the investor chooses the optimal risky asset proportions γ_{jt} . This is independent of wealth and borrowing. In other words, an investor maximizes the ratio of the risk premium to the return opportunity loss that he can incur with probability α ,

$$\max_{\gamma_{jt}} \frac{E_t(R_{t+1}) - r_t}{r_t - q_\alpha(R_{t+1})} \quad (2.4)$$

In the second stage, the investor chooses the scale of the risky portfolio and the amount borrowed b_t based on his budget constraint at time t . Therefore, the optimal borrowing amount can be expressed as

$$b_t = \frac{s - W_t(1 + r_t)}{q_\alpha(R_{t+1}) - r_t} - W_t \quad (2.5)$$

Equation (2.4) and (2.5) give the complete solution to a safety-first investor's problem. One variable, the VaR ($q_\alpha(R_{t+1})$), remains unknown in the two-stage optimization described above. The estimate of VaR is determined by the parameter α and the portfolio distribution. In order to forecast the VaR, we introduce several VaR models in the next section.

C. Value-at-Risk Models

We classify the main existing models into three categories as Manganelli and Engle (2001):

- Parametric (RiskMetrics)
- Nonparametric (Historical Simulation, Monte-Carlo Simulation and Nonpara-

metrically Estimated Distribution)

- Semiparametric (Extreme Value Theory)

The main difference among these approaches is their assumptions on the return distribution. There are critics of each approach, but the focus of this study is to investigate the empirical application of the available methodologies.

1. Model Set-up

Consider a financial return series y_t which follows the process

$$y_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t z_t$$

where $\mu_t = E(y_t|I_{t-1})$, $\varepsilon_t^2 = E(\varepsilon_t^2|I_{t-1})$, and $\{z_t\} \equiv \{\varepsilon_t/\sigma_t\}$ has the conditional distribution function $G_t(z) \equiv Pr(z_t \leq z|I_{t-1})$, I_{t-1} is the information set at time $t - 1$. Given a tail probability $\alpha \in (0, 1)$, $q_t(\alpha)$ is defined as the conditional quantile:

$$F_t(q_t(\alpha)) = \alpha$$

This conditional quantile, also called VaR, can be estimated by:

$$q_t(\alpha) = F_t^{-1}(\alpha) = \mu_t + \sigma_t G_t^{-1}(\alpha)$$

Therefore, a VaR model involves estimating the distribution function F_t or G_t . Several models are briefly described in the next section.

2. Parametric Models

a. RiskMetrics

This method was developed by J.P. Morgan in 1995 to calculate VaR. It has a very simple form and assumes that the return of a portfolio has a conditional normal

distribution,

$$q_t(\alpha) = \mu_t + \sigma_t \Phi^{-1}(\alpha)$$

where $\Phi(\cdot)$ is the standard normal distribution function with $\Phi^{-1}(0.01) = -2.326$ and $\Phi^{-1}(0.05) = -1.645$. The variance σ_t is given recursively by an exponentially weighted moving average (EWMA),

$$\sigma_t^2 = 0.94\sigma_{t-1}^2 + 0.06(y_{t-1} - \mu_t)^2$$

where $\mu_t = \frac{1}{t-1} \sum_{j=1}^{t-1} y_j$.

3. Nonparametric Models

a. Historical Simulation (HS)

Historical simulation assumes that the distribution of returns $\{y_t\}$ remains the same in the past and in the future, and the empirical distribution of historical returns is used in forecasting VaR. The way to estimate the VaR is to arrange the sample in increasing order as

$$y_{(1)} \leq y_{(2)} \leq \dots \leq y_{(n)}$$

Then we know the property of the order statistic y_ℓ which is asymptotically normal with mean x_α and variance $\alpha(1-\alpha)/[nf^2(x_\alpha)]$. That is

$$y_{(\ell)} \sim N\left[x_\alpha, \frac{\alpha(1-\alpha)}{nf^2(x_\alpha)}\right], \quad \ell = n\alpha$$

where n is the sample size, x_α is the α -th quantile of $F(x)$, which is the same as $q_t(\alpha)$, and $f(x_\alpha)$ is the density function of $F(x)$. The advantage of this simple model is that there is no specific distributional assumption. On the other hand, it has many drawbacks. The assumption that the distribution is unchanged over time may be problematic in financial markets. Most important, the use of historical returns makes

it impossible for VaR estimates to be greater than the largest historical loss. This also makes it impossible to make conclusions about probabilities lower than $1/n$ where n is the sample size.

b. Monte-Carlo Simulation (MC)

Unlike the HS method which draws simulated portfolio values from historical data, MC simulation generates a rich set of possible portfolio returns and then estimates VaR from the simulated portfolio returns. For this paper, we assume assets returns are normally distributed. In particular, let the dynamics of $P(t)$ follow geometric Brownian motion with drift:

$$dP(t) = \mu_t P(t) dt + \sigma_t P(t) dW(t)$$

where $W(t)$ is a standard Brownian motion, and μ_t and σ_t are the drift and the volatility parameters, respectively. Thus, the solution to this stochastic differential equation is $P(t) = P(0) \exp([\mu_t - \frac{1}{2}\sigma_t^2]t + \sigma_t W(t))$. Since we estimate next day's VaR, the solution can be rewritten as

$$P(t) = P(t-1) \exp([\mu_t - \frac{1}{2}\sigma_t^2] + \sigma_t z_t),$$

where z_t is simulated from a standard normal distribution. We draw 1,000 values of z_t , and calculate 1,000 values for $P(t)$ from above equation. The empirical α -th quantile of $e_t \equiv \log(P(t)/P(t-1))$ is our VaR estimate, while σ_t^2 is estimated by $\frac{1}{t-2} \sum_{j=1}^{t-1} (e_j - \hat{\mu}_t)^2$ with $\hat{\mu}_t = \frac{1}{t-1} \sum_{j=1}^{t-1} e_j$.

c. Nonparametric Estimation (NP)

The "weighted" Nadaraya-Watson estimator

$$F(y | x_t) = \frac{\sum_{i=1}^n p_i K_h(x_i - x_t) 1(Y_i \leq y)}{\sum_{i=1}^n p_i K_h(x_i - x_t)}$$

where $K_h(\cdot)$ is a kernel function with bandwidth parameter h , $1(\cdot)$ is an indicator function which decides left or right tail behavior to be estimated, and $p_i \equiv p_i(x_t)$ is the weight function that can be regarded as the local empirical likelihood. For our financial market application, we set $(y_t, x_t) = (y_t, y_{t-1})$.

4. Semiparametric Models

a. Extreme Value Theory (EVT)

All the models we discussed above attempt to estimate the the entire distribution of return. Extreme value theory focuses on modelling the tail behavior directly. Three approaches will be discussed here. These are the Generalized Extreme Value (GEV) distribution, the Generalized Pareto distribution (GPD) and the Hill estimator. In all cases, we study the behavior of extreme return values in the sample by applying EVT. The essential difference among the GEV, GPD and Hill approach is how they define an extreme return. GEV divides the sample into m groups, and the minima (or maxima for right tail estimation) in each group are the extreme returns. In the GPD these extreme values are defined as the exceedances over a threshold u . The optimal methods for choosing m or the threshold u are not well defined, but it is important to note that they cannot be determined purely on the basis of statistical theory. The GPD provides VaR calculations that are relatively stable to changes in u , while GEV is very sensitive to the choice of m . Similar to the HS method, we find

order statistics by arranging the sample in increasing order as

$$y_{(1)} \leq y_{(2)} \leq \dots \leq y_{(n)}.$$

Generalized Extreme Value (GEV) Distribution Assume that the returns y_t are serially independent and the range of returns y_t is $[l, u]$. Then the CDF of a normalized minimum $y_{(1*)} \equiv \frac{y_{(1)} - \beta_m}{\alpha_m}$, denoted by $F_*(x)$, is given by

$$F_*(x) = 1 - \exp[-(1 + kx)^{1/k}] \quad \text{if } k \neq 0$$

where $x = y_{(1*)}$.

There are three parameters to estimate: The shape parameter k , the location parameter β_m and the scale parameter α_m . We follow Longin (1996, 2000) and divide the sample into g non-overlapping subsamples. Each subsample contains m observations. We obtain estimates of these three parameters by maximizing the likelihood function over subperiod minima. We calculate the VaR by applying the following equation:

$$q(\alpha) = \beta_m - \frac{\alpha_m}{k} \{1 - [-\ln(1 - \alpha)^m]^k\}$$

We use $m = 10$ when we apply this model in Section 4.

Generalized Pareto Distribution (GPD) After fixing a threshold u , we look at all exceedances e over u . The distribution of extreme values e is given by

$$H(e) = 1 - (1 - \frac{ke}{\delta})^{1/k}$$

where $\{e_i\}_{i=1}^m$ is the sample of exceedances over threshold u , and k and δ are parameters that can be estimated by MLE.

The α -th quantile can then be estimated by

$$q(\alpha) = u - \frac{\delta}{k} \left(1 - \left(\frac{n\alpha}{m}\right)^k\right)$$

In our empirical analysis, we follow Neftçi (2000) and use the empirical 10% quantile as the threshold u .

Hill Estimator In this approach the estimator applies directly to the returns $\{y_t\}_{t=1}^n$. If m denotes the number of negative observations in the sample, the Hill (1975) estimators of k can be defined as

$$\hat{k} = -\frac{1}{\lambda} \sum_{i=1}^{\lambda} (\ln|y_{(m-\lambda-i)}| - \ln|y_{(m-\lambda)}|)$$

where λ is a positive integer. We chose λ as in Bao et al (2006) as the value that includes 1.5% of the sample data. Given an estimate of k , the VaR can be found as

$$q(\alpha) = \left[\frac{m}{\lambda}(1 - \alpha)\right]^{\hat{k}} y_{(\lambda+1)}.$$

D. Empirical Results

1. Data

The particular problem we investigate is to choose the optimal investment strategy when a hypothetical investor has the choice of two international stock index funds. In our analysis, a U.S. investor maximizes his expected daily payoff by allocating his wealth among a risk-free asset, the S&P 500 Index, and the Taiwan Weighted Index. We take the U.S. 3-month Treasury Bill as the risk-free asset. To convert the foreign index return in terms of U.S. dollars we use the exchange rate between the U.S. and Taiwanese dollar. Our sample is a daily data set from January 1, 1991 to December

30, 2005, with a total of 3913 observations². The U.S. and Taiwan return series are given by the log difference of index prices. All of the daily data are retrieved from Datastream.

We split the sample into an in-sample period and an out-of-sample period. The exact procedure is as follows. We begin by estimating the model over the entire in-sample period, using data through the end of 1995. We then begin a rolling window forecasting and portfolio selection procedure. We take a five year rolling window of data to estimate VaR for portfolios consisting of a variety of linear combinations of U.S. and Taiwan equity, and we do so for all seven VaR estimation methods. We do this for all data from January 1, 1996 to December 30, 2005. Then, following the first stage of the safety-first portfolio selection problem, we choose the optimal portfolio composition as stated in equation (2.4) at each date from January 1, 1996 through December 30, 2005. Note that portfolio returns on the risky assets are $R_{t+1} = \gamma_t R_{tw,t+1} + (1 - \gamma_t) R_{us,t+1}$. Investor holds $\gamma_t * 100$ percent of Taiwan stock and $(1 - \gamma_t) * 100$ percent of U.S. stock at time t . The returns of Taiwan stock have already been adjusted for exchange rate. Expectations of $R_{us,t+1}$ are estimated by an AR(1) model, and expectations of $R_{tw,t+1}$ are estimated by an AR(2) model. We describe this in more detail below.

After we choose the optimal portfolio composition for each method at time t , we calculate the optimal borrowing amount based on equation (2.5). The disaster level s is set equal to $0.95W$, so that a disaster is a 1-day 5% decline in wealth. We investigate the case when the tail probability is set so that $\alpha = 0.01$, and also $\alpha = 0.05$. Our empirical findings are shown in the next section.

²We also investigated a longer sample, from January 1, 1988 to December 30, 2005. The results from this longer sample were very similar to the sample we used.

2. Results

a. Preliminary Statistics

Figure 1 plots the out-of-sample stock index levels and returns, and interest rate and exchange rate.

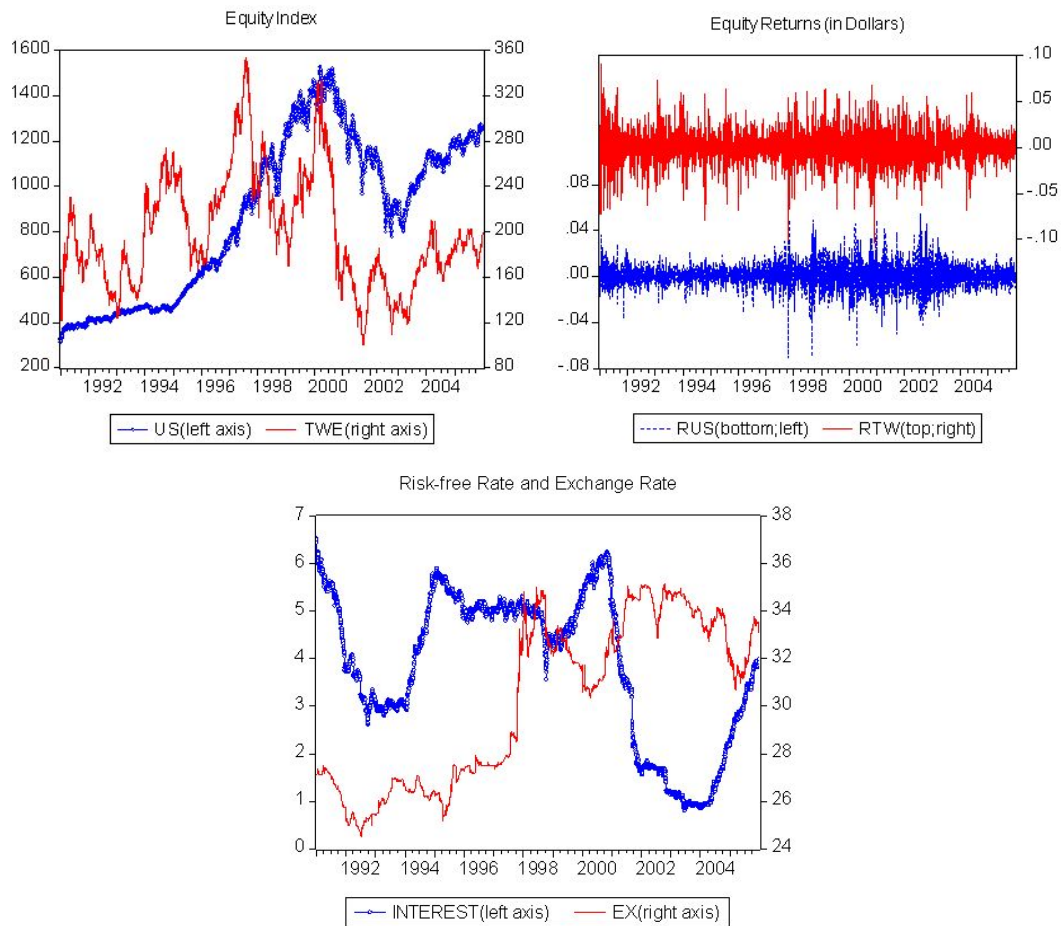


Fig. 1. Preliminary Data

Table I provides descriptive statistics for the S&P 500 Index, the Taiwan Weighted Index, the exchange rate between the U.S and Taiwanese dollar, dollar returns on the S&P 500 and the Taiwan Weighted Index and the U.S. 3-month Treasury Bill rate,

all for our entire period from January 1, 1991 to December 30, 2005, and for our out-of-sample period of January 1, 1996 to December 30, 2005. The average U.S. index return was much higher than the Taiwan index return over this time period. This may be due to the Asian financial crisis, which occurred in 1997. Note that the Taiwan index had more extreme returns, both high and low, than the U.S. index. Therefore the standard deviation of the Taiwan index returns is higher than for the U.S. index. Note too that the exchange rate and interest rate varied substantially during the sample period.

b. Portfolio Selection

Following the procedures described above, we first estimated the VaR by all seven methods for returns of each combination of U.S. and Taiwan indices. We evaluated VaR for both $\alpha = 0.01$ and $\alpha = 0.05$. Recall that $R_t = \gamma_t R_{tw,t} + (1 - \gamma_t) R_{us,t}$. We vary the fraction of Taiwan stock γ_t from 0% to 100% by steps of size 10%. Thus we have 11 portfolios of U.S. and Taiwan stock. We estimate each portfolio's daily VaR using the seven different methods outlined above. Since the results for the two different tail probabilities are similar, we only provide a detailed report for the case $\alpha = 0.01$. We present a brief discussion of the case $\alpha = 0.05$.

Table II presents information on how VaR estimates vary across methods. For presentation purposes we present three portfolios, one invested 100% in U.S. equity, one invested 100% in Taiwanese equity, and one invested 50% in each country. Over 1996-2005 period, the GPD and Hill methods had relatively low VaR, so they appear to be the more conservative methods. The MC and RiskMetrics methods had higher average VaR and seem to be the more aggressive risk management methods.

The first stage of decision making for a safety-first investor is to choose the optimal risky asset proportions γ_t . As stated in Equation (2.4), the particular problem

Table I. Summary Statistics

| Whole Sample Period (01/01/1991- 12/30/2005) | | | | | | |
|---|-----------|----------|----------|-----------|-----------|----------|
| | US | TWE | EX | RUS*100 | RTW*100 | i*100 |
| Mean | 874.5757 | 200.0593 | 30.2004 | 0.0340 | 0.0046 | 0.0104 |
| Median | 933.4750 | 188.7461 | 31.0900 | 0.0087 | 0.0000 | 0.0116 |
| Maximum | 1527.4500 | 354.3946 | 35.1600 | 5.5732 | 9.0588 | 0.0176 |
| Minimum | 311.4900 | 99.5741 | 24.5100 | -7.1127 | -10.4608 | 0.0022 |
| Std. Dev. | 352.7122 | 49.2849 | 3.4342 | 0.9968 | 1.7342 | 0.0042 |
| Skewness | -0.0460 | 0.6310 | -0.0660 | -0.0961 | -0.0865 | -0.4995 |
| Kurtosis | 1.5798 | 2.9794 | 1.3780 | 7.2265 | 5.8786 | 1.9768 |
| Jarque-Bera | 330.3318 | 259.8344 | 431.8869 | 2919.1830 | 1356.2470 | 333.5238 |
| Probability | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Observations | 3914 | 3914 | 3914 | 3914 | 3914 | 3914 |
| Out-of-Sample Period (01/01/1996- 12/30/2005) | | | | | | |
| | US | TWE | EX | RUS*100 | RTW*100 | i*100 |
| Mean | 1087.206 | 204.1638 | 32.1661 | 0.0271 | 0.0021 | 0.0097 |
| Median | 1117.975 | 191.4081 | 32.8000 | 0.0066 | 0.0000 | 0.0118 |
| Maximum | 1527.450 | 354.3946 | 35.1600 | 5.5732 | 6.7922 | 0.0168 |
| Minimum | 598.4600 | 99.5741 | 27.1500 | -7.11 | -10.46 | 0.0022 |
| Std. Dev. | 221.4747 | 54.0789 | 2.4108 | 1.1341 | 1.6820 | 0.0046 |
| Skewness | -0.30 | 0.5373 | -0.87 | -0.09 | -0.14 | -0.35 |
| Kurtosis | 2.4301 | 2.6613 | 2.5694 | 6.1707 | 5.7657 | 1.5604 |
| Jarque-Bera | 73.6473 | 138.0637 | 347.1303 | 1096.842 | 840.2282 | 278.3262 |
| Probability | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Observations | 2610 | 2610 | 2610 | 2610 | 2610 | 2610 |

Table II. Estimated VaR with $\alpha = 1\%$

| $\gamma = 0$ (US stock return) | | | | |
|----------------------------------|--------|-------|---------|--------|
| | mean | std | min | max |
| Riskmetrics | -2.711 | 1.057 | -6.661 | -1.194 |
| HS | -2.671 | 0.594 | -3.451 | -1.558 |
| MC | -2.465 | 0.654 | -3.341 | -1.387 |
| Cai | -2.619 | 0.589 | -3.348 | -1.549 |
| GEV | -2.667 | 0.603 | -3.520 | -1.562 |
| GPD | -4.661 | 0.929 | -6.139 | -2.667 |
| Hill | -3.186 | 0.647 | -4.123 | -1.930 |
| $\gamma = 0.5$ (50% US & 50% TW) | | | | |
| | mean | std | min | max |
| Riskmetrics | -2.521 | 0.875 | -5.918 | -1.016 |
| HS | -2.741 | 0.356 | -3.180 | -2.242 |
| MC | -2.463 | 0.350 | -3.012 | -1.910 |
| Cai | -2.702 | 0.386 | -3.604 | -1.488 |
| GEV | -2.665 | 0.325 | -3.232 | -2.059 |
| GPD | -4.527 | 0.452 | -5.177 | -3.551 |
| Hill | -3.234 | 0.279 | -3.552 | -2.560 |
| $\gamma = 1$ (TW stock return) | | | | |
| | mean | std | min | max |
| Riskmetrics | -4.075 | 1.429 | -10.602 | -1.638 |
| HS | -4.828 | 0.403 | -5.845 | -4.242 |
| MC | -4.186 | 0.358 | -4.838 | -3.616 |
| Cai | -4.714 | 0.372 | -5.843 | -4.121 |
| GEV | -4.558 | 0.248 | -5.235 | -3.876 |
| GPD | -8.023 | 0.547 | -9.240 | -6.107 |
| Hill | -5.743 | 0.375 | -6.395 | -4.754 |

becomes

$$\max_{\gamma_t} \frac{E_t(R_{t+1}) - r_t}{r_t - q_{0.01}(R_{t+1})} \quad (2.6)$$

$$\text{where } E(R_{t+1}) = \gamma_t E(R_{tw,t+1}) + (1 - \gamma_t) E(R_{us,t+1})$$

We use the rolling five year window to estimate $E(R_{tw,t+1})$ and $E(R_{us,t+1})$ at time t for each data point from 1996-2005. The autoregressive coefficients are re-estimated each period and then used to forecast. For each method, each day, we have 11 values for $\frac{E_t(R_{t+1}-r_t)}{E_t(r_t)-q_{0.01}(R_{t+1})}$, corresponding to the 11 portfolios. The portfolio giving the highest value is the optimal risky portfolio choice for that day.

Figure 2 shows each method's daily optimal risky portfolio choice for γ_t from January 1, 1996 to December 30, 2005. For example, in the top left graph, for the RiskMetrics method, there is one dot for each time period, and the dot appears at the mark on the vertical axis that indicates the portfolio mix that is optimal for that day.

The optimal choice of the portfolio mix, γ_t , does not vary as much as we expected across VaR estimation methods. In fact, for 63% of the days, the seven methods all chose the same portfolio. For less than 3% of the days, the seven optimal portfolio returns did not move in the same direction, i.e., all positive or all negative returns. There is not a single day that the seven methods led to seven different portfolio combination.

Table III shows the distribution of optimal portfolio choice for three specific portfolio combinations: U.S. stocks only, Taiwan stocks only, and an equally weighted combination of the two. The first column indicates that, using the RiskMetrics method, on 54% of days we would choose $\gamma_t = 0$, and on 19% of days we would choose $\gamma_t = 1$. We would choose $\gamma_t = 0.5$ only 3% of the days. Thus an safety-first

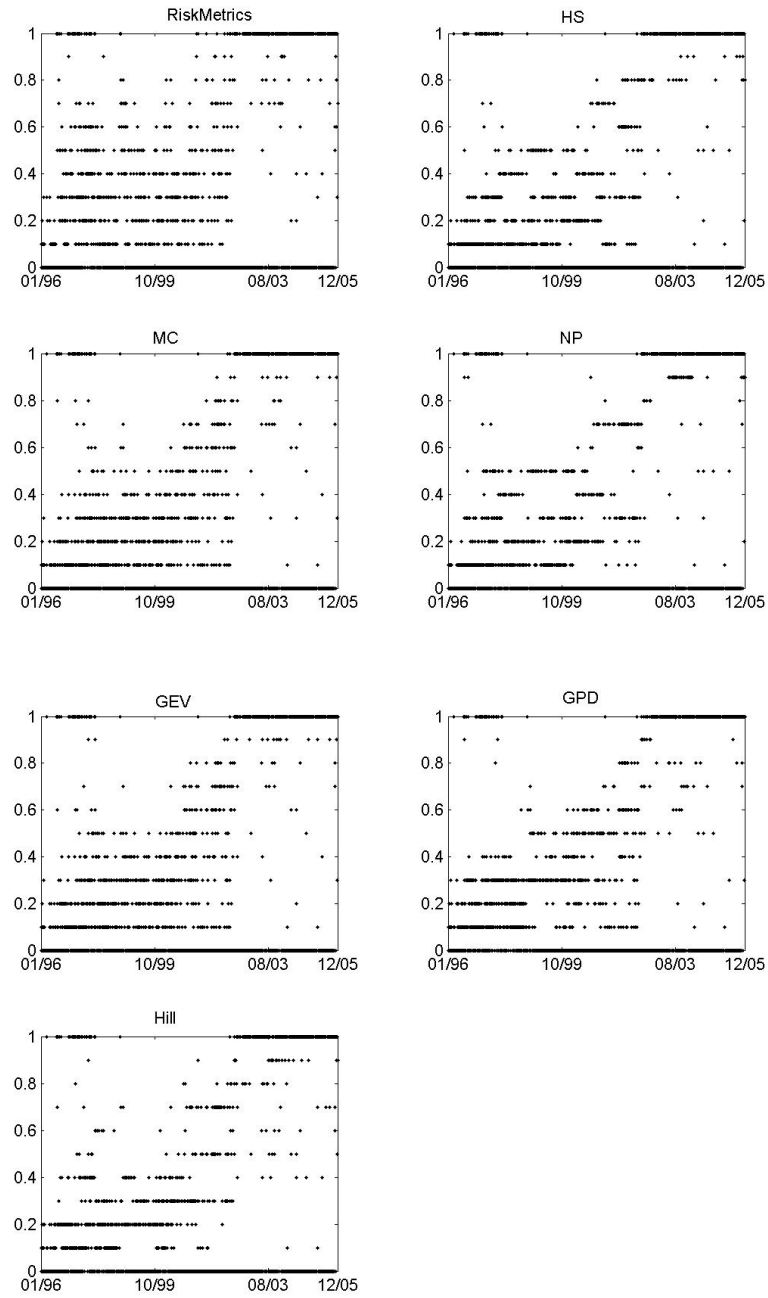


Fig. 2. Optimal Portfolio Choice: $\alpha = 1\%$

investor using RiskMetrics would invest more frequently in U.S. stock, since over this period U.S. stocks had a higher average return and lower risk than Taiwan stocks.

Note that the other six methods also invested heavily in U.S. stocks. Interestingly, the GPD method gives the lowest percent of days with $\gamma_t = 0$, but this method still chooses $\gamma_t = 0$ on 49.7% of the days.

Table III. Optimal Portfolio Choice: $\alpha = 1\%$

| | RiskMetrics | HS | MC | NP | GEV | GPD | Hill |
|-----------|-------------|-------|-------|-------|-------|-------|-------|
| gamma=0 | 0.543 | 0.524 | 0.552 | 0.522 | 0.543 | 0.497 | 0.530 |
| gamma=0.5 | 0.030 | 0.023 | 0.015 | 0.034 | 0.020 | 0.030 | 0.015 |
| gamma=1 | 0.188 | 0.181 | 0.184 | 0.173 | 0.183 | 0.180 | 0.173 |

Figure 3 plots the optimal risky portfolio's corresponding VaR for each method over time. VaR values are in percentage terms. The pattern of VaR estimated by RiskMetrics seems to differ markedly in the graphs from the other six methods, and the six other methods seem to show a similar pattern although with different levels of VaR as indicated by the different scales on the vertical axis.

In the second stage, our investor chooses the optimal amount of borrowing, b_t . For computational purposes we normalize W to 1, and set s to $0.95W$. For the case of the tail probability $\alpha = 0.01$, our investor can be described as first wanting the probability that tomorrow's loss will be greater than 5% to be less than 1%. Then, given that this requirement is met, he maximizes his expected return. Equation (2.5) which describes optimal borrowing, becomes,

$$b_t = \frac{0.95 - (1 + r_t)}{q_{0.01}(R_{t+1}) - r_t} - 1 \quad (2.7)$$

The evolution of optimal borrowing is shown in Figure 4, and is also characterized

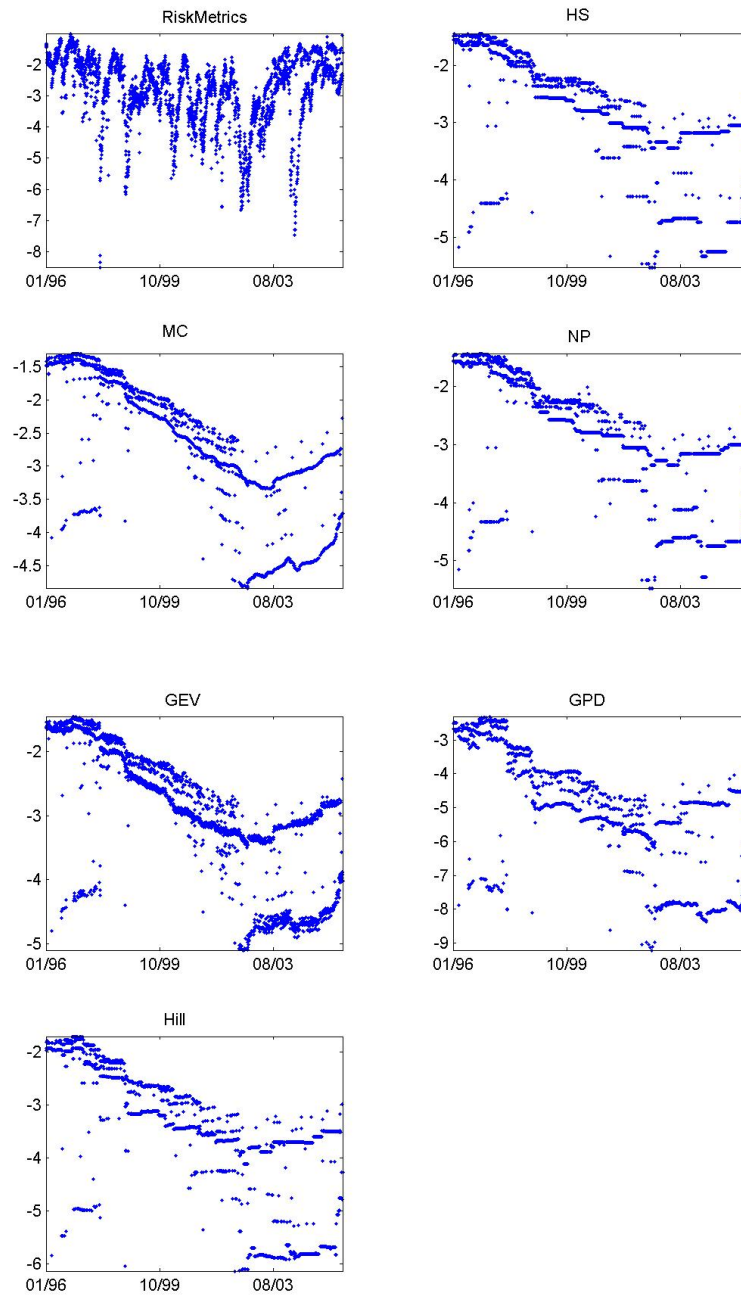


Fig. 3. Optimal VaR Evolution: $\alpha = 1\%$

Table IV. As we saw before, the GPD and Hill methods appear to be more conservative, since an investor using these methods would borrow less than an investor using the other methods. The MC and RiskMetrics methods are more aggressive, and investors are leveraged more heavily. Note there is a decreasing trend of b after a certain point, which occurs about the time of the Asian financial crisis. We also see that the pattern of borrowing with RiskMetrics seems different than the other six methods.

Table IV. Optimal Borrowing $b : \alpha = 1\%$

| | RiskMetrics | HS | MC | NP | GEV | GPD | Hill |
|--------|-------------|-----------|----------|-----------|-----------|----------|----------|
| Mean | 1.0197 | 0.92143 | 1.12 | 0.96661 | 0.91777 | 0.11259 | 0.60028 |
| Std | 0.72572 | 0.69866 | 0.80331 | 0.70968 | 0.66264 | 0.40401 | 0.55439 |
| Median | 0.92141 | 0.78009 | 0.86354 | 0.78597 | 0.7166 | 0.015217 | 0.4468 |
| Min | -0.4105 | -0.095113 | 0.033494 | -0.085907 | -0.019939 | -0.45677 | -0.18525 |
| Max | 3.9183 | 2.4397 | 2.8139 | 2.481 | 2.4275 | 1.1596 | 1.9058 |

c. Return of Portfolio

One of the reasons we carry out the safety-first model is to compare the performance of different VaR models in a simulated real investment environment. Figure 5 and Table V presents the returns on our optimal risky portfolio from the seven different methods. Because the optimal portfolio proportions among the two markets are not very different across the seven methods, the daily returns on the risky portfolios are also close among the methods.

The main disparity across methods is in the optimal amount of borrowing. Different estimates of VaR lead to different borrowing amounts, different leverage, and this has important implications for portfolio risk and return. As a result, we expect

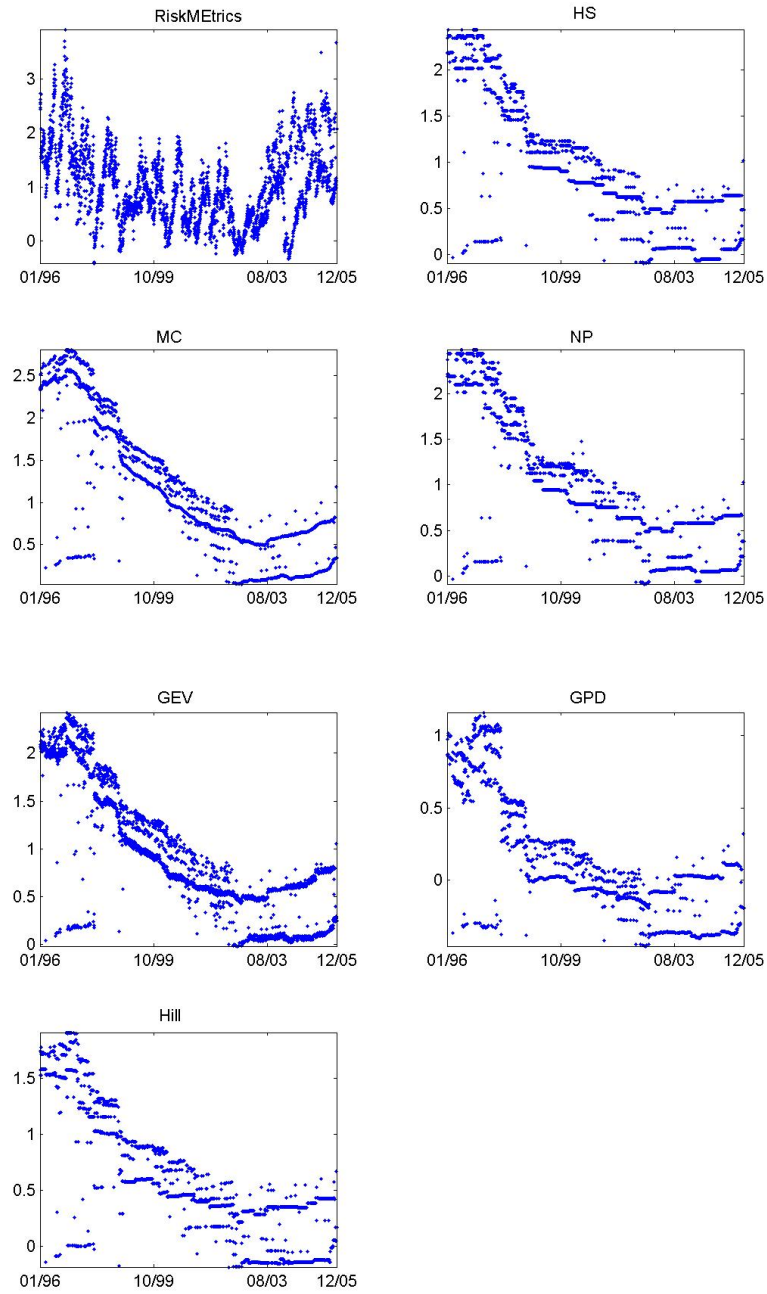


Fig. 4. Optimal Borrowing $b : \alpha = 1\%$

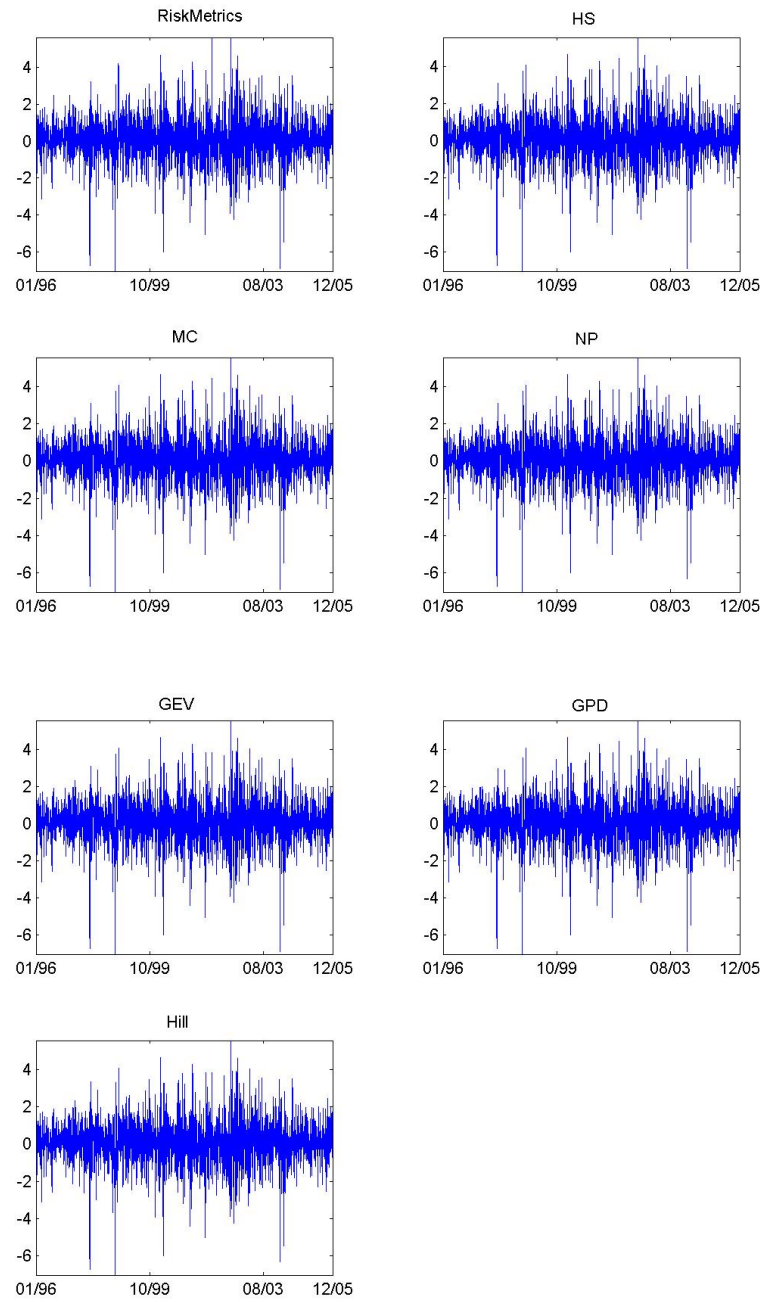


Fig. 5. Returns on Risky Portfolio: $\alpha = 1\%$

Table V. Risky Portfolio Return (in percentage): $\alpha = 1\%$

| | RiskMetrics | HS | MC | NP | GEV | GPD | Hill |
|--------|-------------|--------|--------|--------|--------|--------|--------|
| mean | 0.058 | 0.054 | 0.057 | 0.054 | 0.054 | 0.054 | 0.055 |
| std | 1.178 | 1.180 | 1.179 | 1.174 | 1.177 | 1.172 | 1.173 |
| median | 0.033 | 0.044 | 0.036 | 0.042 | 0.037 | 0.036 | 0.034 |
| min | -7.044 | -7.044 | -7.044 | -7.044 | -7.044 | -7.044 | -7.044 |
| max | 5.613 | 5.573 | 5.573 | 5.573 | 5.573 | 5.573 | 5.573 |

the overall portfolio returns to be distinct across methods. Figure 6 and Table VI illustrate this phenomenon. The average daily return varies from 0.06% to 0.11%. A striking result is that during some periods the overall portfolio performance was extraordinarily bad for investors using several of the different VaR estimators. During the Asian financial crisis in 1998, on August 31st, the daily return on U.S. stock was -7.04% , and the Taiwan stock return was -2.62% . But seven VaR estimation methods would have had an investor completely in the U.S. stocks at this time, because the Taiwan stock market experienced negative returns near this date. However, this day was a greater catastrophe for safety-first investors that the individual market returns would indicate. In fact, only two of the VaR methods resulted in portfolios with losses of less than 10%. The other five methods all had at least a 13% loss on that day. The reason for this was mainly due to the level of borrowing chosen by each method. Under RiskMetrics estimation of VaR, the optimal borrowing amount on August 31, 1998 was 0.31, which is 31% of initial wealth W . The corresponding value for GPD was 0.18, and b equals 1.35, 1.75, 1.36, 1.29 and 0.91, for HS, MC, NP, GEV and Hill estimation, respectively. The HS, MC, NP, GEV and Hill methods all chose a large amount of borrowing to invest in risky assets, and therefore incurred huge losses when the risky portfolio suffered a decline. The overall portfolio return of

the HS, MC, NP and GEV methods was calculated as, respectively, -16.5% , -19.1% , -16.6% , and -13.5% . Thus, by choosing portfolios based on VaR estimated using these methods, an investor would have lost more than 13% of his wealth in a single day.

Table VI. Overall Portfolio Return: $\alpha = 1\%$

| | RiskMetrics | HS | MC | NP | GEV | GPD | Hill |
|-----------------|-------------|---------|---------|---------|---------|---------|---------|
| Arithmetic Mean | 0.0009 | 0.0009 | 0.0011 | 0.0010 | 0.0010 | 0.0006 | 0.0008 |
| Std. | 0.0204 | 0.0214 | 0.0238 | 0.0219 | 0.0213 | 0.0120 | 0.0177 |
| Median | 0.0006 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0004 | 0.0005 |
| Min | -0.1008 | -0.1654 | -0.1912 | -0.1663 | -0.1617 | -0.0832 | -0.1349 |
| Max | 0.0687 | 0.0876 | 0.1004 | 0.0888 | 0.0851 | 0.0474 | 0.0715 |

Figure 7 plots the investor's wealth evolution when an investor starts with initial wealth 1 dollar at the beginning of 1996³. The upper graph in Figure 8 plots wealth evolution following MC, RiskMetrics, HS and GPD all in one graph, while the bottom one plots evolution following the other three methods and MC method. Although NP moves closely to MC in Figure 8, MC does better than any other methods during our sample period. The probabilities⁴ of MC exceeding other methods are 96%, 99%,

³For example, from July 1998 to August 1998, there was a huge decline in the wealth for most of methods. If we pick up a particular day, say July 21, 1998, follow RiskMetrics, the optimal borrowing amount is 1.9 and the investor invests 20% in Taiwan stock and 80% in U.S. stock. On that day, the return on U.S. equity is -0.016, the return on Taiwan equity is -0.012 and the interest rate is 0.00013. The wealth from July 20, 1996 is 3.28, which is the initial wealth on July 21. Therefore, the wealth of investor on July 21 can be calculated as $(3.28 + 1.9 * 3.28) * [0.2(1 - 0.012) + 0.8(1 - 0.016)] - 1.9 * 3.28 * (1 + 0.00013)$, which equal to 3.14. That is a 4% decrease in wealth within one day.

⁴The probability for RiskMetrics is calculate as the ratio of the number of times that wealth following MC method is greater than wealth following RiskMetrics over total number of trading days. Probability for other methods are computed in the same way.

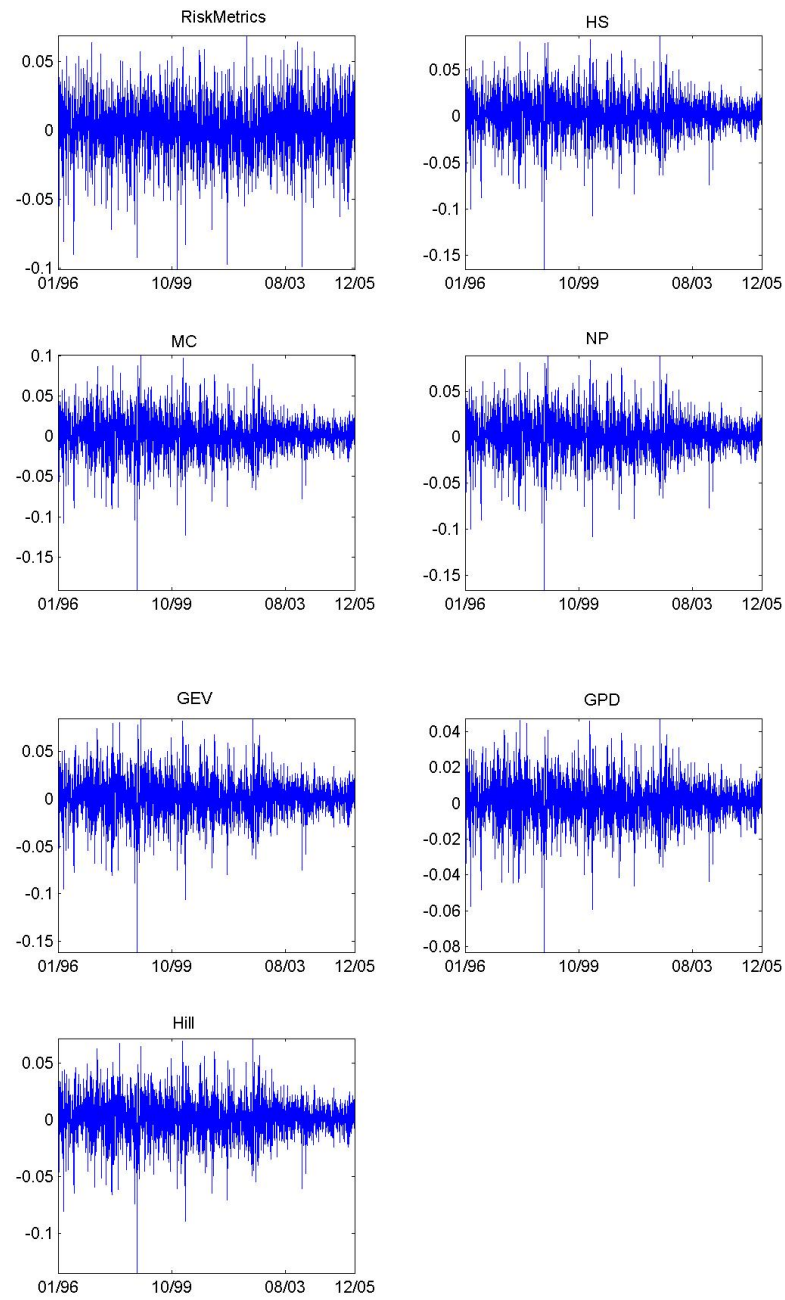


Fig. 6. Returns on Overall Portfolio: $\alpha = 1\%$

91%, 96%, 98% and 96% for RiskMetrics, HS, NP, GEV, GPD, and Hill methods, correspondingly.

d. Properties of Portfolio Return

Our concern, and the concern of a safety-first investor, is the performance of portfolios chosen based on the seven VaR estimation methods. Figure 9 and Table VII illustrate how the extreme values of the portfolio return are distributed among the different methods. In Table VII we report the percent of time that each VaR estimation method resulted in the largest(and, separately, smallest) daily return. More than ninety percent of the daily maximum portfolio returns were from one of three methods: RiskMetrics, MC or GPD. The same is true of the minimum returns. Figure 9 shows this information graphically.

Table VII. Distribution of Extreme Values of Portfolio Return with $\alpha = 1\%$

| | RiskMetrics | HS | MC | NP | GEV | GPD | Hill |
|----------|-------------|------|------|------|------|------|------|
| minValue | 0.24 | 0.01 | 0.25 | 0.03 | 0.01 | 0.44 | 0.02 |
| maxValue | 0.25 | 0.01 | 0.26 | 0.03 | 0.01 | 0.43 | 0.01 |

There is another interesting finding. During our forecasting days associated with positive returns on the risky portfolio, the GPD method gave the minimal returns 86.2% of the time among the seven methods, while RiskMetrics and the MC method provided the maximal returns on 91.6% of the days. During the days when there were negative returns, the GPD method gave the best of these negative returns 87.7% of time. On the other hand, the probability that the RiskMetrics or the MC method turned out to be the minimal return was 84.6%. Thus the odds for positive returns from these methods was only slightly higher than for negative returns during our

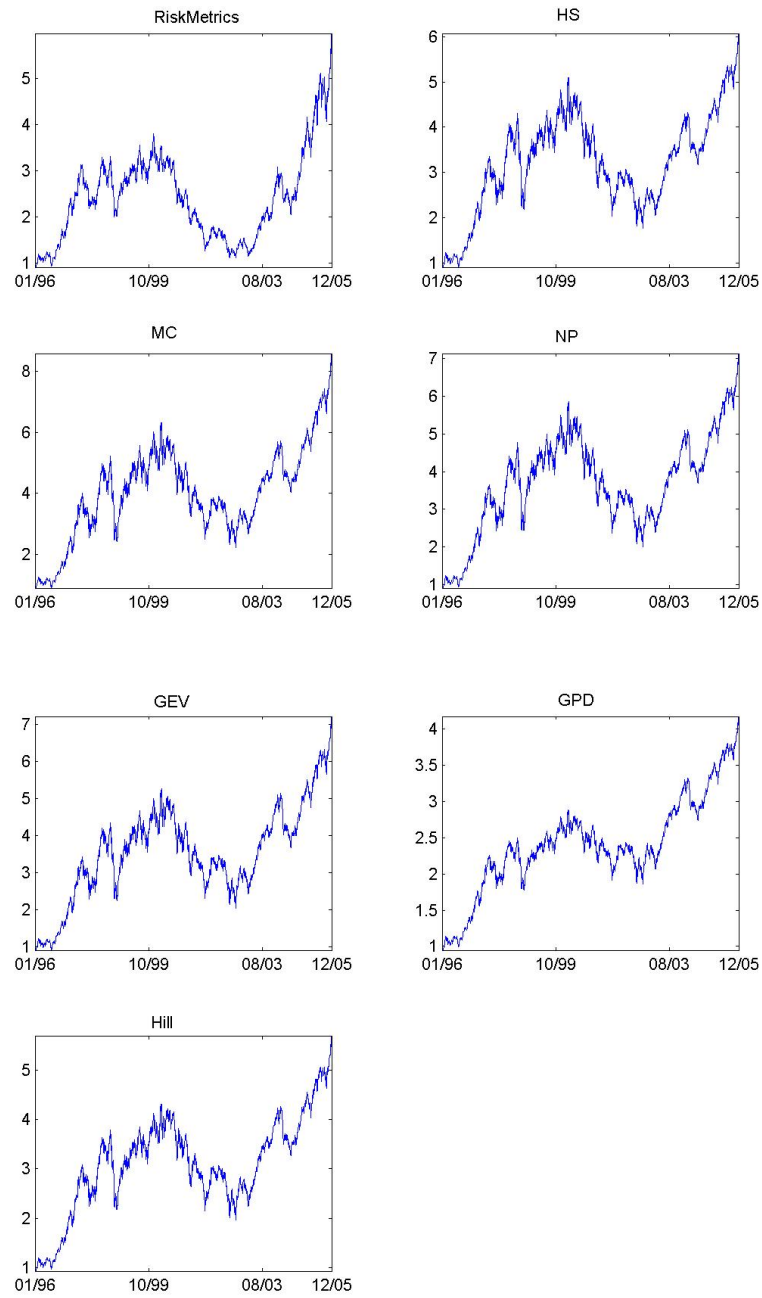


Fig. 7. Investor's Wealth Evolution: $\alpha = 1\%$

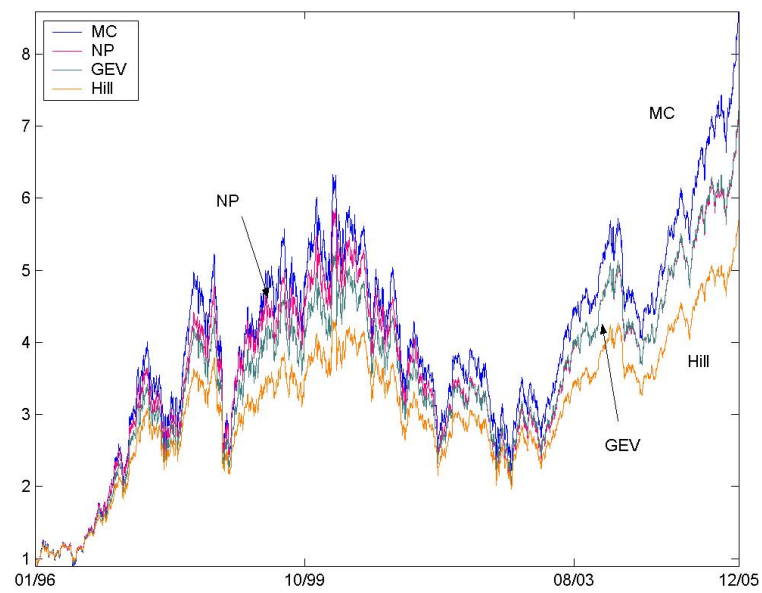
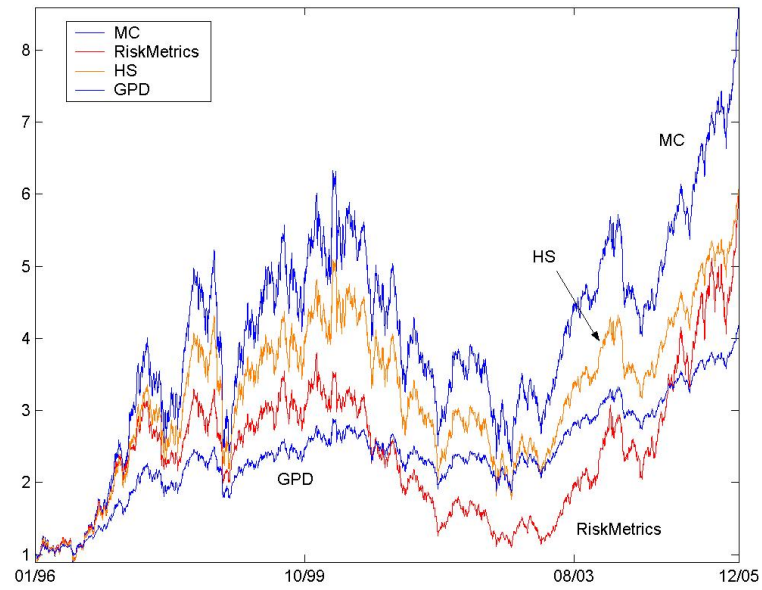


Fig. 8. Investor's Wealth Evolution 2: $\alpha = 1\%$

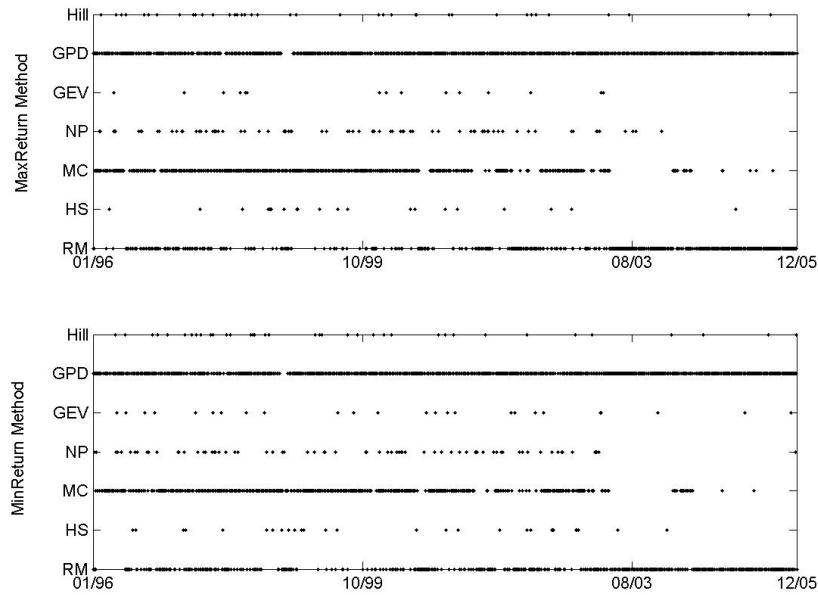


Fig. 9. Extreme Value of Returns on Overall Portfolio: $\alpha = 1\%$

forecasting period, January 1996 to December 2005. Hence, these three methods give us much of the available information about the range of portfolio returns in the sample period.

e. Results for the Case Tail Probability = 5%

Figure 10 and Figure 11 show the evolution of the optimal portfolio choice and the optimal borrowing for the case of the tail probability $\alpha = 5\%$. Figure 12 and Figure 13 plot the returns of Risky portfolio and overall portfolio when $\alpha = 5\%$. When we redo the exercise for the case $\alpha = 5\%$, the main difference from the case $\alpha = 1\%$ is that the optimal borrowing amounts for every method are much higher. Table VIII shows the results. In this case, investors would invest a higher fraction of their wealth in the risky assets, because these investors now insist only on being 95%, instead of

99% confident that tomorrow's loss will not be higher than 5%. Consequently, these investors would choose a more aggressive investment strategy, which translates to greater borrowing. As shown in Table IX, the overall portfolio returns are higher than for the case $\alpha = 1\%$. Figure 14 provides the information on the extreme values of the portfolio returns. As in the previous case, the RiskMetrics and the GPD methods provide much of the information about the boundary of portfolio returns among the seven methods.

Table VIII. Optimal Borrowing b with $\alpha = 5\%$

| | RiskMetrics | HS | MC | NP | GEV | GPD | Hill |
|--------|-------------|---------|---------|---------|---------|---------|---------|
| Mean | 2.2572 | 2.1697 | 1.9545 | 2.1884 | 2.4969 | 0.56431 | 1.5286 |
| Std | 1.1665 | 1.205 | 1.1331 | 1.2146 | 1.2833 | 0.55194 | 0.91019 |
| Median | 2.1018 | 1.7175 | 1.596 | 1.7341 | 2.0874 | 0.37802 | 1.4996 |
| Min | -0.04718 | 0.58933 | 0.42149 | 0.60406 | 0.62643 | -0.1936 | 0.11177 |
| Max | 6.8922 | 4.8361 | 4.3389 | 4.8853 | 5.107 | 1.8633 | 3.6698 |

Table IX. Overall Portfolio Return with $\alpha = 5\%$

| | RiskMetrics | HS | MC | NP | GEV | GPD | Hill |
|-----------------|-------------|---------|---------|---------|---------|---------|---------|
| Arithmetic Mean | 0.0014 | 0.0015 | 0.0015 | 0.0016 | 0.0017 | 0.0008 | 0.0010 |
| Std. | 0.0329 | 0.0357 | 0.0332 | 0.0360 | 0.0397 | 0.0172 | 0.0287 |
| Median | 0.0009 | 0.0007 | 0.0008 | 0.0007 | 0.0008 | 0.0005 | 0.0008 |
| Min | -0.1624 | -0.2925 | -0.2675 | -0.2965 | -0.3347 | -0.1269 | -0.2424 |
| Max | 0.1111 | 0.1532 | 0.1403 | 0.1540 | 0.1814 | 0.0696 | 0.1182 |

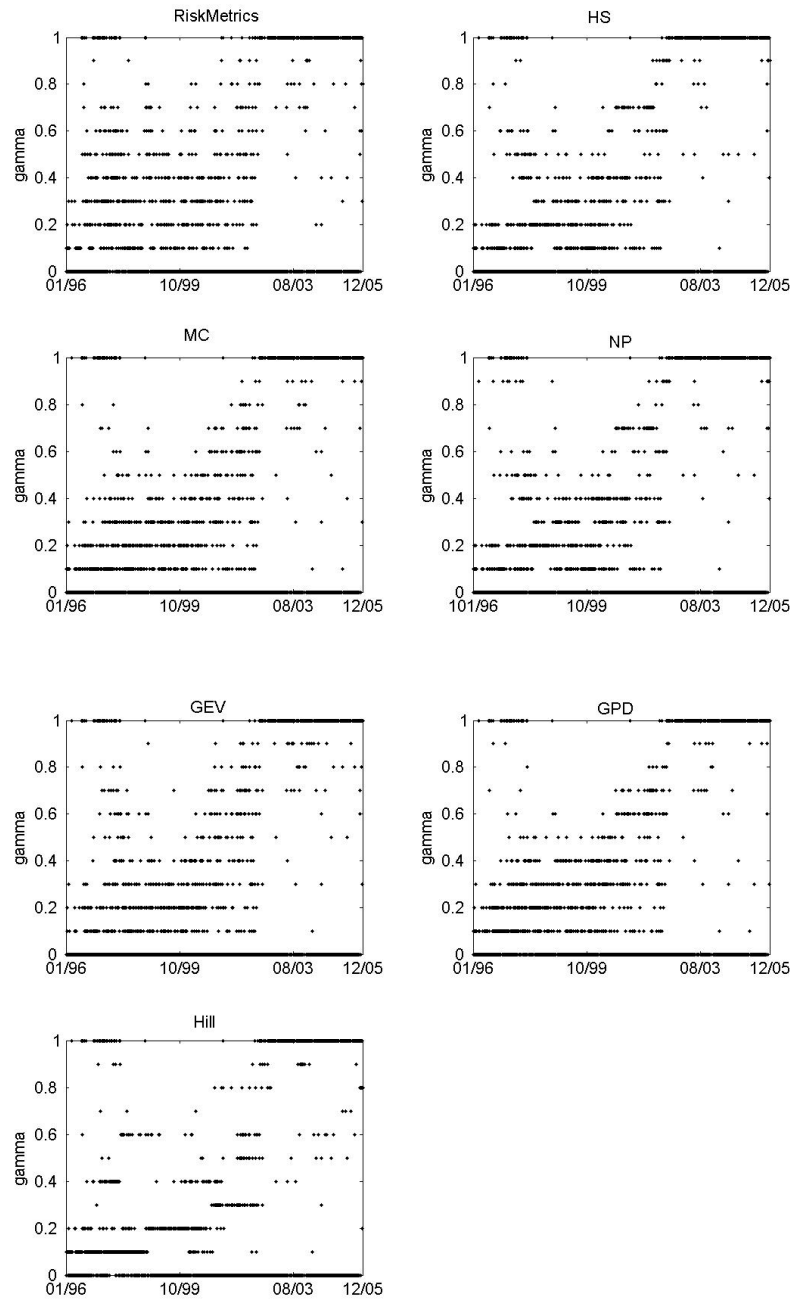


Fig. 10. Optimal Portfolio Choice: $\alpha = 5\%$

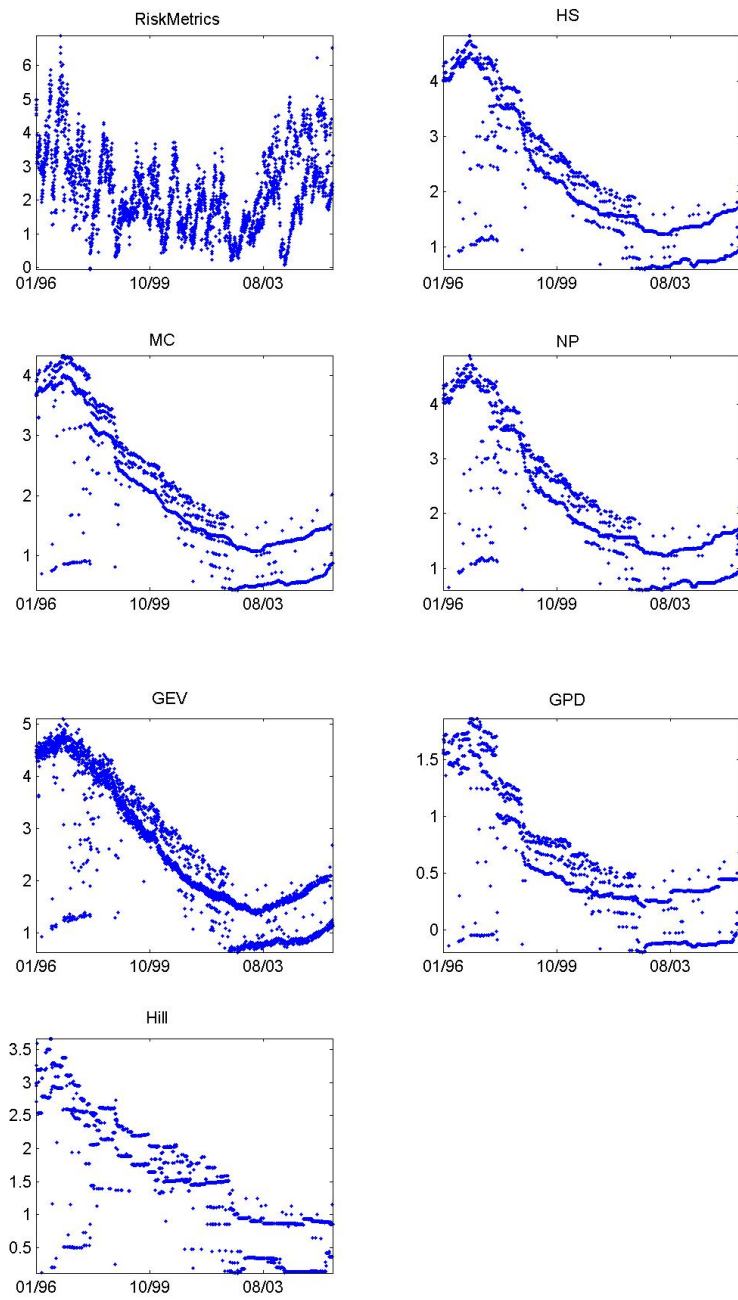


Fig. 11. Optimal Borrowing b : $\alpha = 5\%$

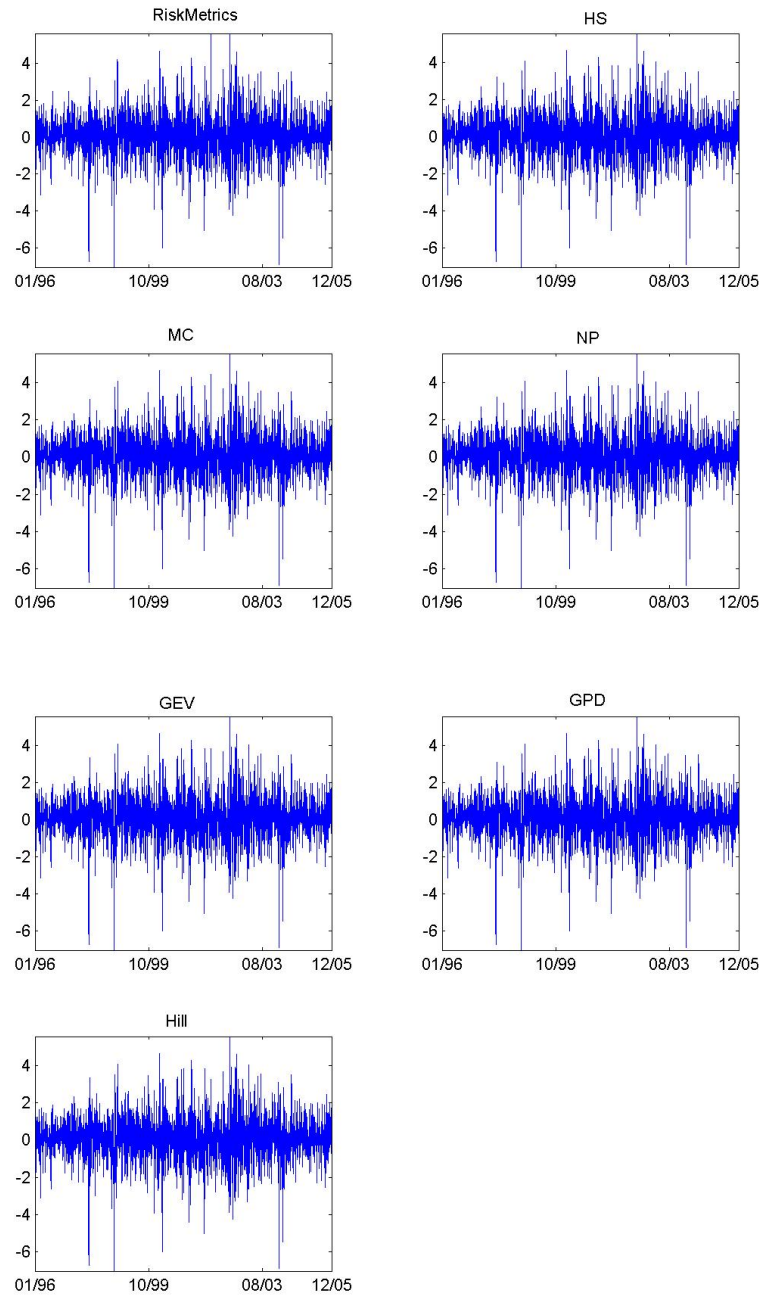


Fig. 12. Returns on Risky Portfolio: $\alpha = 5\%$

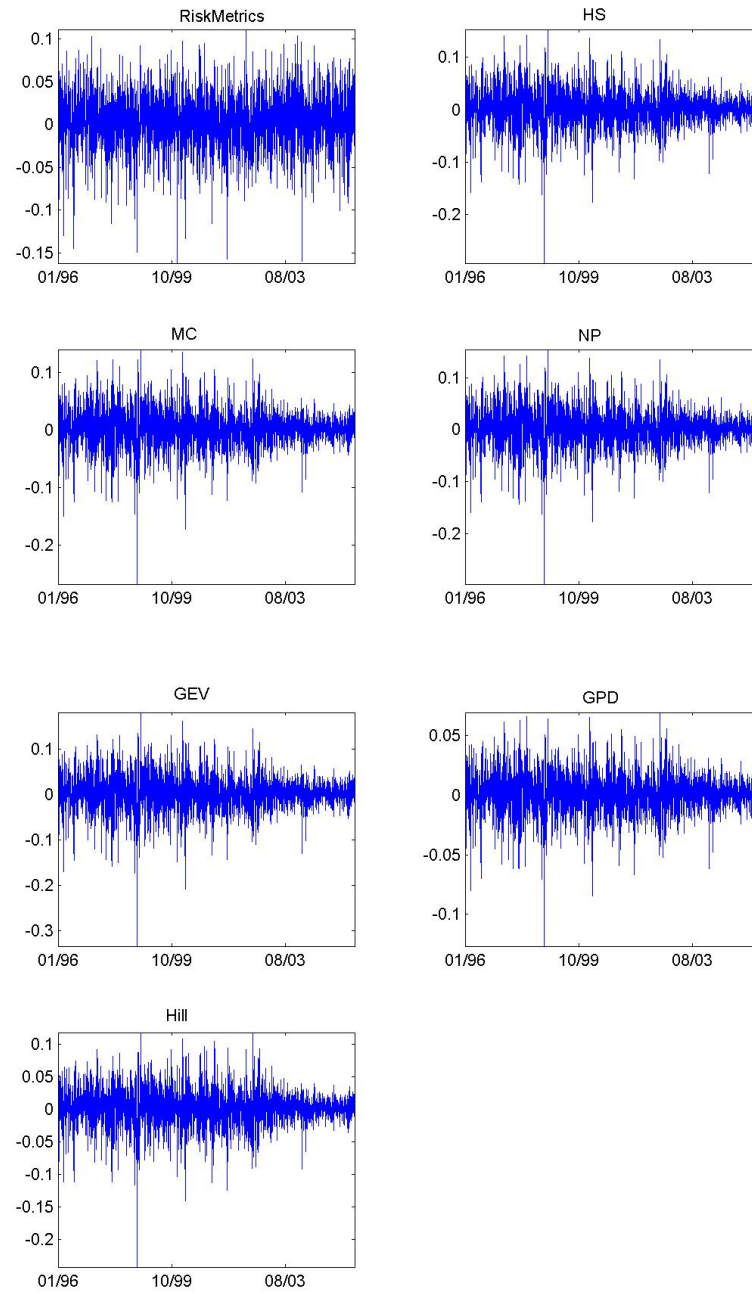


Fig. 13. Returns on Overall Portfolio: $\alpha = 5\%$

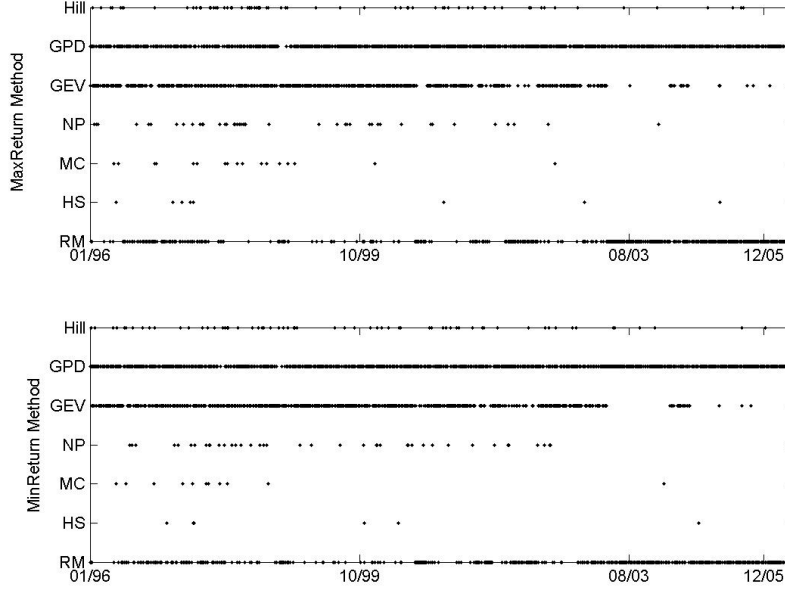


Fig. 14. Extreme Value of Returns on Overall Portfolio: $\alpha = 5\%$

f. Coverage Probability

In Table X, we provide the empirical coverage probability for both cases, $\alpha = 1\%$ and $\alpha = 5\%$. Because the VaR constraint actually helps an investor reduce the downside risk of his portfolio, we look at how often an investor's wealth would fall below the disaster level. The empirical coverage probability is the frequency of violations (i.e., the frequency that the next day's wealth is lower than 95% of initial wealth). For $\alpha = 1\%$ and $\alpha = 5\%$, 5 out of 7 methods overstated the coverage probability. Only the GPD and Hill method had violations less than 1%, and the frequency of violations for the GPD method is much lower than 1%, but the frequency of MC method is much higher than 2%. We note, however, that it is not clear that the method with the best tail coverage probability is the best model. For a “real-world” portfolio choice

problem, such as the one analyzed in this paper, it is likely that investors want to maximize returns after satisfying some type of safety constraint. This suggests looking beyond the method which gives the best tail coverage probability. The superior performance of the MC method as judged by portfolio returns over this period is a case in point.

Table X. Coverage Probability

| | RiskMetrics | HS | MC | NP | GEV | GPD | Hill |
|-------------------|-------------|--------|--------|--------|--------|--------|--------|
| $\alpha = 1\%$ | | | | | | | |
| Num of Violations | 35.00 | 44.00 | 59.00 | 45.00 | 42.00 | 3.00 | 21.00 |
| Frequency | 0.0134 | 0.0169 | 0.0226 | 0.0172 | 0.0161 | 0.0011 | 0.0080 |
| $\alpha = 5\%$ | | | | | | | |
| Num of Violations | 152.00 | 165.00 | 142.00 | 163.00 | 199.00 | 20.00 | 107.00 |
| Frequency | 0.0582 | 0.0632 | 0.0544 | 0.0625 | 0.0762 | 0.0077 | 0.0410 |

E. Conclusions

This paper provides a detailed analysis of the performance of seven Value-at-Risk models in practice. Since safety-first portfolio selection is a useful application of VaR models, we demonstrate safety first portfolio selection guiding by various VaR models. Using daily data for U.S. and Taiwan stocks for the period 1996 to 2005, we illustrate how an investor can build up different portfolios from two stock indices and a risk-free asset. Following different VaR estimation methods, we found safety-first investor will choose significantly different amounts of borrowing. Thus, the scale of the risky portfolio and the amount borrowed is diverse across methods. The daily overall portfolio returns are distinct over the period we studied. Some of the methods' protection performances are not satisfactory in face of crisis.

CHAPTER III

DYNAMICS RELATION BETWEEN VOLUME AND PRICES: MIXTURE NORMAL ESTIMATION

A. Introduction

A large literature investigates the role of volume in equity markets. The evident has been found by enormous empirical studies that volume plays an important role in markets. Karpoff (1987) provides a review of previous research on the contemporaneous correlation between price changes and trading volume. A strong positive relation between the magnitude of the price change and volume is reported in his survey. There are also numerous papers¹ concerned with the relationship between volume and volatility of stock returns. Our focus is on the dynamics relation between return and trading volume. For example, Campbell, Grossman, and Wang (1993) and Llorente, Michaely, Saar, Wang (2002) investigate the serial correlation of stock returns to the level of volume.

However, why price changes are related to volume in some ways and how volume gets involved in markets are not definitive. In this paper we try to determine how the volume related to the behavior of prices and properties of the asset. On the modelling side, our paper is related to Blume, Easley and O'Hara (1994)(henceforth BEO) who try to answer these questions under the rational expectations framework. Their work follows the standard approach that some fundamental value of stock is unknown to all traders and traders receive signals containing the information of the fundamental. The traders do not know the price at which their order will be executed

¹For example, theoretical work by Brown (1989) and Wang (1994) and empirical work by Koopman (2005) and Lee (2002).

before the trade occurs. And they try to find the Walrasian equilibrium with single one equilibrium price and volume in each period. In their model, the aggregate supply is fixed. This assumption makes the source of noise comes solely from the quality of the information. Volume, therefore, contains some unique information on information quality.

In this paper, we propose a model in which stock prices may not attain their fundamental value, although the prices exhibit tendencies towards the fundamental value. The market process in this model can be taken as moving from one inefficiency to another. Hence, there are more than one transaction price for each trading period in our model. At each transaction price, the number of buy order shares may not equal to the number of sell order shares. Another important difference between our model and model of BEO(1994) is each trader's expectation on price is different in our model. Traders begin with common beliefs about the risky asset's payoff, but the private signals received by them cause their beliefs to diverge. Investors decide both prices at which orders are executed and the amount of demand for risky asset. This means that we consider a pure limit order market in this model. There is no market order or specialist involved. Based on both common and private information available to the traders, they will generate their optimal limit orders. Transaction occurs only when the limit price is attained, which means the execution may not carry out for every order. Our construction of model allows us to look at how real world market functions.

B. Simple Version of BEO(1994) Model

Our starting point is investigating simplified version of BEO(1994) model. BEO(1994) begins with the standard rational expectations model(see Grossman and Stiglitz

(1980)) in which a finite set of investors, indexed by $i = 1, \dots, I$, can trade a risk-free asset and a risky asset. Assume investor i has zero endowment of risky asset before trade and some risk-free asset n_{i0} . The investor's problem is to maximize his negative exponential utility functions in each period after he receive a private signal, y_{it} , on the value of the risky asset. The choice variable of the investor is the demand on risky asset d_{it} . The fundamental value of the risky asset is given by the random variable ψ , where ψ is normally distributed with mean ψ_0 and variance $1/\rho_0$. We only analyze time 1 problem for BEO model. Trader i 's problem can be stated as

$$\begin{aligned} \max_{d_i} E_i(U|H^i) &= E\{-\exp[-\omega_i]|H^i\} \\ \text{subject to} \quad \omega_i &= d_i(\psi - p) + n_{i0} \end{aligned}$$

where ω_i is trader i 's terminal wealth which depends on the payoffs of both risky asset and risk-free asset and the trading decision. H^i is trader i 's individual information set, which includes the private signal y_i and a conjecture of the price of the risky asset p . y_i is given by

$$y_i = \psi + e_i$$

where the distribution of each e_i is $N(0, 1/\rho)$. The average signal is $\bar{y} = \sum_{i=1}^I y_i/I$, and it converges to ψ with probability 1 as the number of traders grows large. In BEO model, the aggregate supply of risky asset X is fixed. At equilibrium per capital supply, $x = X/I$, equal per capital demand,

$$x = \sum_{i=1}^I d_i/I.$$

Although the traders do not know at which price the trade will take place, they have a common belief that p is a linear function of aggregate information (\bar{y}) and per

capital supply (x):

$$p = \alpha\psi_0 + \beta\bar{y} - \gamma x.$$

With this assumption, they solve the Walrasian equilibrium price and volume. Trader i 's demand for risky asset is

$$d_i = \frac{E(\psi|H^i) - p}{Var(\psi|H^i)} = \rho_0(\psi_0 - p) + \rho(y_i - p).$$

Since there is no endowment of risky asset, the equilibrium requires $\sum_{i=1}^I d_i = 0$. The market clearing price is derived as

$$p = \frac{\rho_0\psi_0 + \rho\bar{y}}{\rho_0 + \rho} = \frac{\rho_0\psi_0 + \rho\psi}{\rho_0 + \rho}$$

The second part of equation holds when the number of traders I goes to infinite. In fact, the BEO model divides the traders into two groups. The traders from different groups receive signals from different distributions. The precision of the first group's signals is known only to the traders from that group. In this simplified BEO model, the above construction is same as saying ρ is not a common knowledge to all the traders. Only a group of people know the value of ρ . Those traders who do know real value of ρ can reveal true value of risky asset ψ with observed equilibrium price. However, from the price alone, the traders without information of ρ cannot discern what the fundamental value is.

BEO model shows that if traders observe both the price and the volume, then even the traders having no previous knowledge on ρ can reveal ψ . Here volume contains information on precision of signals ρ . Blume et al. define the volume as one half of the sum of the absolute values of demands at equilibrium price.

$$V = \frac{1}{2} \frac{1}{I} \sum_{i=1}^I |\rho_0(\psi_0 - p) + \rho(y_i - p)|$$

Blume et al. solve for the equilibrium volume which is a function of price p , precision of signals ρ and fundamental value ψ . Therefore, the group of traders without knowing ρ can reveal it from volume. The fundamental value is discovered for both groups.

In their model, volume is defined in the way that the demand is equal to the supply of risky asset. This assumption is only an approximation to workings of the market². Even the authors themselves pointed out that the market is a dynamic process and real markets are never in a Walrasian equilibrium. Another issue in their model is that even traders who use limit orders do not know the price at which their order will execute before the trade occurs. In our model, our target is not looking at a single equilibrium price but many transaction prices. At each transaction price, demand and supply of risky asset may not equal. The market price is related to volume not as simple as BEO model. The excess demand of asset drives the prices. Our model is introduced in the next section.

C. The Basic Model

The setup of our model is standard. There are finite number of investors in the market. The fundamental value of the stock is unknown to the traders, but the distribution of the fundamental value is public information. Every trader also receives his private signal on the fundamental value, which equals to fundamental value plus an error term. They start with some endowments of riskless asset and risky asset before trades take place. Every period, based on his information set (common information and private signal on value of the risky asset) each investor allocates his wealth between the risky and risk-free asset in order to maximize his utility. There are no close-form solutions to the market state at the end of each trading period. The simulation

²See Benink (2001) for more information on equilibrium in financial markets

method is used to solve the problem. The detailed description of simulation is in Su (2007). He developed a platform called MiniExchange. In this platform, the traders put their limit orders at the beginning of each period. After matching their orders, order may be filled, or canceled, or stay in the limit order book to the next period. The unexecuted part of the partially filled orders can stay in the limit order book to next period. However, if the trader submits new order next period, the remaining unexecuted part will be canceled and the new order will be added in the book.

We begin by describing our model with a single round of trade. There is a single risky asset in the market. A set of I traders indexed by $i = 1, \dots, I$ possess random endowments x_{i0} of the risky asset and n_{i0} of the risk-free asset. The endowments are i.i.d. Each trader receives a private signal y_{it} on the value of the risky asset at period t . The trader's problem is to maximize his negative exponential utility function at time t given all the information available at t . His choice variables include the amount of risk-free asset, limit order size and its price. If $n_{i,t-1}$ and $x_{i,t-1}$ are the number of units of the risk-free asset and number of shares of stock trader i endows from last period, the budget constraint facing by him is $d_{it}l_{it} = n_{i,t-1} - n_{it}$, where d_{it} is the size of limit order and l_{it} is the prespecified price of limit order. $d_t > 0$ denotes limit buy; $d_t < 0$ denotes limit sell³. For example, if trader i wants to put a limit buy at time t , which means he wants to hold more stocks than before, the cost of purchasing excess demand of stocks is $d_{it}l_{it}$. This cost comes from cash holding from last period, and anything left becomes next period's cash holding. Suppose $P(l_t)$ is the probability of limit-order execution and $F(l_t)$ is its CDF. The trader's problem now can be stated

³We do not consider short sell in our model, which means if $d_t < 0$, absolute value of d_t need to be no more than x_{t-1} . Traders cannot sell more shares than he owns. This is a reasonable assumption, because some countries' security markets forbid equity short and even those markets which allow it have strict restriction on short positions.

as:

$$\begin{aligned}
\max_{n_{it}, d_{it}, l_{it}} E_{it}(U|H_t^i) &= \max_{n_{it}, d_{it}, l_{it}} E_{it}\{-\exp[-\omega_{it}]\}H_t^i\} \\
&= E\{-\exp[-(n_{it} + (d_{it} + x_{i,t-1})\psi)]\}F(l_t) \\
&\quad + E\{-\exp[-(n_{i,t-1} + x_{i,t-1}\psi)]\}[1 - F(l_t)] \\
&\text{subject to} \quad n_{i,t-1} = n_{it} + d_{it}l_{it}
\end{aligned}$$

where H_t^i is trader i 's information set at time t , and ψ is the stock's fundamental or liquidation value. Traders have heterogenous beliefs on stock's liquidation value, which can be represented by a normal distribution $N(\psi_0, 1/\rho_0)$. $H^i = (\psi_0, \rho_0, \rho, y_i)$, where y_i is private signal received by trader i .

$$y_{it} = \psi + e_{it}$$

where the distribution of each e_{it} is $N(0, 1/\rho)$.

The solutions of trader i 's problem at t are shown as follows.

$$d_{it} = \frac{E(\psi|H_t^i) - l_{it}}{\text{Var}(\psi|H_t^i)} - x_{i,t-1} \quad (3.1)$$

$$\begin{aligned}
&-\exp[-C_{i,t-1} + d_{it}l_{it} - (d_{it} + x_{i,t-1})E(\psi|H_t^i) + \frac{(d_{it} + x_{i,t-1})^2 \text{Var}(\psi|H_t^i)}{2}][f(l_t) + d_{it}F(l_t)] \\
&+ \exp[-C_{i,t-1} - x_{i,t-1}E(\psi|H_t^i) + \frac{x_{i,t-1}^2 \text{Var}(\psi|H_t^i)}{2}]f(i, l_t) = 0
\end{aligned}$$

equivalent to

$$\exp[d_{it}l_{it} - d_{it}E(\psi|H_t^i) + \frac{d_{it}(d_{it} + 2x_{i,t-1})\text{Var}(\psi|H_t^i)}{2}][f(l_t) + d_{it}F(l_t)] - f(l_t) = 0 \quad (3.2)$$

From Bayesian analysis of normal distribution, conditional on H^i , trader i views ψ as normally distributed with mean $E(\psi|H_t^i) = \frac{\psi_0\rho_0 + y_{it}\rho}{\rho + \rho_0}$, and conditional variance $\text{Var}(\psi|H_t^i) = (\rho + \rho_0)^{-1}$.

In order to study the the dynamics of the prices and trading volume in our model, multiple period transactions are involved. It is difficult to find a closed form solution for such a complicated problem, hence simulation method is necessary. We only briefly report the simulation results of our model in this chapter. Please check Su (2007) for details on simulation procedure.

D. Simulation Results

The model's simulation results predict that volume has some predictive power on the price movement in the future. The explanation is naturally rising from the model's set up. For example, if we suppose there is a positive shock to the fundamental value of the stock, on average, more traders receive higher private signals when the shock first takes place. However, traders cannot distinguish whether this is due to a change of fundamental value or error of his own signal. So the best response is to adjust his limit order price, but not fully to the change of his private signal. Since most of the traders will have expectation on fundamental value higher than last transaction price, the total shares of buy orders outnumber sell orders, and the trading volume stays at a relatively low level. Eventually, the price will go up. In such case, low trading volume implies a big price increase in the near future. That's how the trading volume has predictive power over the price. Similarly, when there is negative information flow, the current volume stays at a relatively low level, but there will be a large decrease in the future price.

Another simulation result predicts that volume is positively related to the magnitude of the price change. This result is consistent with many empirical findings and has been highly documented. In our framework, stock price gradually reach up to its equilibrium level after shocks. During the process, the more adjustment taking place

in price, the less unbalance between buy and sell orders, therefore, more transactions. In the next section, the prediction of our model has been verified by our empirical study.

E. Empirical Results

1. Data and Sample Description

a. Daily Returns and Volume

Our empirical data consists of IBM stock return and volume series during the period from 3 January 1994 to 9 December 2005. The investigation period constitutes a total of 3009 observations. We obtain daily data for prices, number of shares traded and number of shares outstanding from Datastream. We detrend the volume as Liorrente et al (2002). Daily turnover is defined as the ratio of number of shares traded to the number of shares outstanding on that day. We use daily turnover as the measure of trading volume. We then take logs of the turnover. A small constant (0.00000255) is added to the turnover before taking logs to avoid the problem of zero daily trading volume. To detrend the log turnover series, we subtract a 200-trading-day moving average of log turnover. Figure 15 shows the time series of returns and volume we use for our empirical analysis.

$$V_t = \logturnover_t - \frac{1}{200} \sum_{s=-200}^{-1} \logturnover_{t+s}$$

where $\logturnover_t = \log(turnover_t + 0.00000255)$.

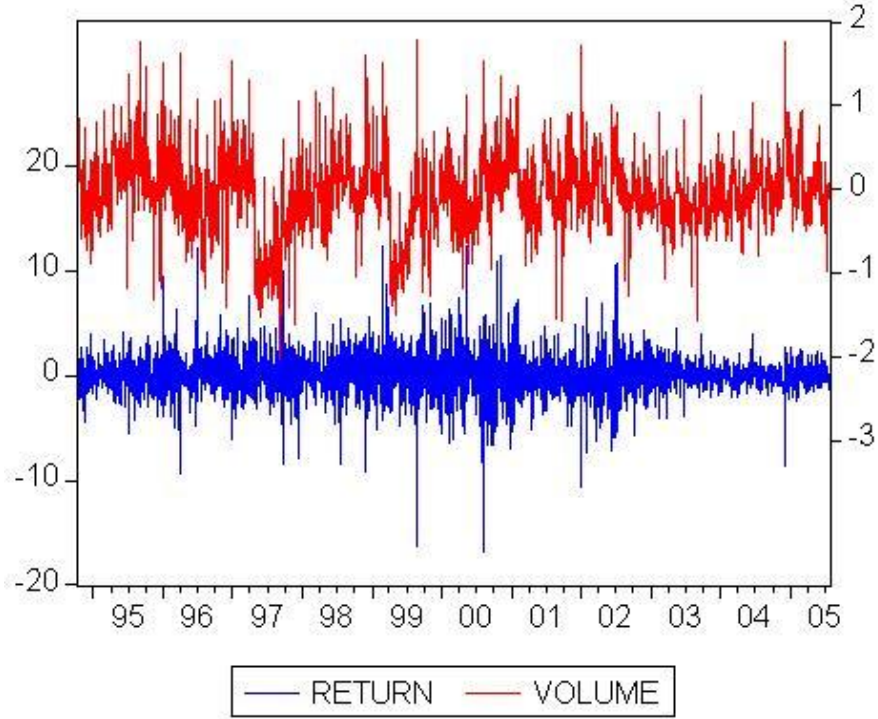


Fig. 15. IBM Returns and Detrended Volume

2. Estimation

a. Tests of the Dynamic Volume-Return Relation

We define $\{r_t\}$ as the underlying daily return series. Volume series $\{v_t\}$ are defined as our normalized volume, the detrended log turnover. Table XI presents the results of IBM stock for time-series regression which estimates the relation between return and volume. The coefficients on the volume or the lags of volume are not significant in Table XI. We believe the result showing that return and volume are not related is misleading, since the volume-return relation can be opposite under different circumstances. So we run another regression and analyze the relation between the absolute value of return and volume. It turns out the absolute value of return is

Table XI. Least Square Estimation: Return

| $r_t = \alpha + \sum_{j=1}^4 \beta_j r_{t-j} + \sum_{k=1}^4 \gamma_k v_{t-k} + \delta v_t + \varepsilon_t$ | | | | |
|--|-------|----------------------|-------|-------------|
| return | Coef. | Std. Err. | t | Probability |
| β_1 | -.037 | .019 | -1.94 | 0.052 |
| β_2 | -.012 | .019 | -0.65 | 0.518 |
| β_3 | -.006 | .019 | -0.32 | 0.748 |
| β_4 | .038 | .019 | 2.01 | 0.045 |
| γ_1 | .198 | .11 | 1.74 | 0.082 |
| γ_2 | .032 | .11 | 0.28 | 0.779 |
| γ_3 | -.097 | .11 | -0.85 | 0.396 |
| γ_4 | .094 | .11 | 0.89 | 0.372 |
| δ | -.12 | .11 | -1.15 | 0.250 |
| α | .062 | .040 | 1.55 | 0.121 |
| Adj $R^2 = 0.0017$ | | Number of obs = 2804 | | |

highly correlated with volume. The estimation results are presented in Table XII.

b. A Mixture Normal Model

To capture the relation between returns and volume under different information flows, which as our model suggests, we apply mixture normal distribution to individual stock returns. It was first introduced by Clark (1973) using mixture normal density to model the return series. The model is further developed by Epps and Epps (1976) and Tauchen and Pitts (1983). Chung (2005) also use this approach to estimating the effect of price limits. This model includes a latent information inflow s_t which affects stock returns. We then assume r_t is independently and mixture normally distributed.

Table XII. Least Square Estimation: Absolute Value of Return

| $ r_t = \alpha + \sum_{j=1}^4 \beta_j r_{t-j} + \sum_{k=1}^4 \gamma_k v_{t-k} + \delta v_t + \varepsilon_t$ | | | | |
|--|-------|---------------------|-------|-------------|
| abs. return | Coef. | Std. Err. | t | Probability |
| β_1 | .052 | .019 | 2.76 | 0.006 |
| β_2 | .12 | .019 | 6.53 | 0.000 |
| β_3 | .13 | .019 | 7.01 | 0.000 |
| β_4 | .15 | .019 | 8.30 | 0.000 |
| γ_1 | -.42 | .077 | -5.45 | 0.000 |
| γ_2 | -.34 | .078 | -4.38 | 0.000 |
| γ_3 | -.22 | .077 | -2.79 | 0.005 |
| γ_4 | -.32 | .071 | -4.48 | 0.000 |
| δ | 1.66 | .067 | 24.92 | 0.000 |
| α | .81 | .052 | 15.71 | 0.000 |
| Adj $R^2 = 0.2346$ | | Number of obs =2804 | | |

The stock returns conditional on s_t is assumed to be normal, that is,

$$r_t \mid s_t \sim N(\mu(s_t), \sigma^2(s_t) \mid s_t)$$

For simplicity, we also assume $\{r_t\}$ is identically distributed. Therefore, the unconditional density of r_t is still identical and can be given by

$$f(r_t) = \int f(r_t \mid s_t) g(s_t) ds_t,$$

where $g(\cdot)$ is the density for s_t .

In our analysis, we have three states of information arrival: strong negative

information arrival ($s_t = -$), insignificant information arrival ($s_t = 0$) and strong positive information arrival ($s_t = +$). Let $P(s_t = -) = p_-$, $P(s_t = 0) = p_0$ and $P(s_t = +) = 1 - p_- - p_0$. The conditional distribution of r_t at state $s_t = i$ is given by

$$f(r_t | s_t = i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left\{-\frac{(r_t - \mu_i)^2}{2\sigma_i^2}\right\},$$

where μ_i and σ_i^2 ($i = -, 0, +$) represent the conditional means and variances.

The unconditional density function then is given by

$$\begin{aligned} f(r_t) &= \sum_{i=-,0,+} f(r_t, s_t = i) \\ &= \frac{p_-}{\sqrt{2\pi}\sigma_-} \exp\left\{-\frac{(r_t - \mu_-)^2}{2\sigma_-^2}\right\} + \frac{p_0}{\sqrt{2\pi}\sigma_0} \exp\left\{-\frac{(r_t - \mu_0)^2}{2\sigma_0^2}\right\} \\ &\quad + \frac{p_+}{\sqrt{2\pi}\sigma_+} \exp\left\{-\frac{(r_t - \mu_+)^2}{2\sigma_+^2}\right\} \end{aligned} \quad (3.3)$$

In our empirical study, we want to reveal the relation between stock return and volume. The estimation equation between returns and volume we investigate is

$$r_t = \alpha + \sum_{j=1}^n \beta_j r_{t-j} + \sum_{k=0}^m \gamma_k v_{t-k} + \varepsilon_t, \quad (3.4)$$

where ε_t satisfies a normal mixture. Now the conditional means of each state can be described as following

$$\mu_i = \alpha^i + \sum_{j=1}^n \beta_j^i r_{t-j} + \sum_{k=0}^m \gamma_k^i v_{t-k} \quad (3.5)$$

where i indicates different state. The parameters in equation (3.4) need to be estimated are probability of information arrival p_i , conditional mean μ_i and conditional variance σ_i^2 . In the process of estimating μ_i , we need to estimate $\{\alpha\}$, $\{\beta\}$, and $\{\gamma\}$. $\{\gamma\}$ shows how returns and trading volume are related. All of the parameters can be

Table XIII. Summary Statistics (1994-2005)

| | RETURN | VOLUME |
|--------------|----------|-----------|
| Mean | 0.055462 | -0.051765 |
| Median | 0.027615 | -0.042943 |
| Maximum | 12.366 | 1.7922 |
| Minimum | -16.891 | -8.7274 |
| Std. Dev. | 2.102768 | 0.487841 |
| Skewness | 0.00252 | -2.049278 |
| Kurtosis | 9.419504 | 38.92323 |
| Jarque-Bera | 4823.294 | 153005.8 |
| Probability | 0 | 0 |
| Observations | 2809 | 2809 |

estimated using standard maximum likelihood techniques.

We then apply this methodology to investigate the case of IBM stock, where the optimal lags are $n = 4$ and $m = 4$. We use Eviews to compute the optimal lags. Table XIII provides the descriptive statistics for the returns and the detrended volume on IBM stock in sample period.

Table XIV ⁴ summarizes the evidence on the return-volume relation under different states. The first ten coefficients estimate the return-volume relation when there is positive information flow. The coefficients on the one-day-ahead return and the four-day-ahead return are both negative and statistically significant. Current return and current volume are highly correlated since the coefficient on the current volume

^{4***} indicates the estimates are significant at 1% level; ** indicates the estimates are significant at 5% level; * indicates the estimates are significant at 10% level.

is 5.25 with standard error of 0.36. The results are consistent with the prediction of our model. After a positive shock in the market, the stock price is expected to go up in the near future, so a large positive return is expected. As we explained earlier, the volume remains at a relatively low level in the process of market adjustment to the fundamental value. Therefore, our empirical findings support our model prediction that the lags of volume are negatively correlated to current return.

The estimation results under a negative information flow are also consistent with our model prediction. The estimates are shown as the third ten coefficients. The coefficients on the one-day-ahead return and the four-day-ahead return are 1.01 and 0.65, correspondingly, and both of them are statistically significant. This is the opposite case from the one under positive information flows. Due to the mismatch on buy orders and sell orders, the volume will stay at a lower level when negative shocks first take place in the market. We expect the price will drop in the near future after the negative shock. Therefore, lower volume is associated with lower returns in the future, which indicates that the lags of volume and current return are positively related. The estimation results in both positive information and negative information case are consistent with our model predictions.

The dynamic relation between volume and prices is not significant in the case of no strong information flows, shown as the second ten coefficients in Table XIV. But the simultaneous volume-return relation is 0.27 in this case and statistically significant⁵.

Our empirical work also captures the well-known phenomena that simultaneous volume and absolute value of price changes are positively related. The evidence is

⁵We can tell the different cases under different information flows by looking at the estimated constants under each condition. Positive constant indicates the case of positive information flow; negative constant indicates negative information flow case

Table XIV. Mixture Normal Estimation

| | coeff | Std. Err. | t-ratio | | coeff | Std. Err. | t-ratio |
|----------------------|-----------|-----------|---------|--------------|------------|-----------|---------|
| α^+ | 1.808*** | 0.141 | 12.798 | γ_1^+ | -0.909** | 0.370 | -2.455 |
| β_1^+ | -0.146*** | 0.043 | -3.361 | γ_2^+ | -0.312 | 0.458 | -0.680 |
| β_2^+ | -0.103** | 0.0477 | -2.157 | γ_3^+ | -0.235 | 0.401 | -0.585 |
| β_3^+ | -0.0776* | 0.0468 | -1.659 | γ_4^+ | -1.0064*** | 0.339 | -2.968 |
| β_4^+ | -0.0274 | 0.0468 | -0.584 | γ_0^+ | 5.251*** | 0.363 | 14.484 |
| α^0 | 0.178*** | 0.0653 | 2.729 | γ_1^0 | 0.178* | 0.0934 | 1.906 |
| β_1^0 | -0.0508** | 0.0230 | -2.207 | γ_2^0 | -0.055 | 0.098 | -0.562 |
| β_2^0 | -0.055** | 0.0262 | -2.105 | γ_3^0 | -0.033 | 0.098 | -0.338 |
| β_3^0 | 0.0107 | 0.0369 | 0.2901 | γ_4^0 | 0.0498 | 0.0888 | 0.561 |
| β_4^0 | 0.0172 | 0.0332 | 0.518 | γ_0^0 | 0.274*** | 0.106 | 2.572 |
| α^- | -1.278*** | 0.103 | -12.398 | γ_1^- | 1.011*** | 0.274 | 3.692 |
| β_1^- | -0.138*** | 0.039 | -3.4394 | γ_2^- | 0.549* | 0.296 | 1.855 |
| β_2^- | -0.0068 | 0.0421 | -0.163 | γ_3^- | -0.0024 | 0.318 | -0.008 |
| β_3^- | -0.044 | 0.0411 | -1.065 | γ_4^- | 0.6498** | 0.261 | 2.490 |
| β_4^- | 0.0575 | 0.044 | 1.301 | γ_0^- | -4.271*** | 0.255 | -16.733 |
| $x+$ | 1.463*** | 0.148 | 9.9098 | | | | |
| <i>Probability</i> + | 0.188*** | | | | | | |
| $\sigma+$ | 2.711*** | 0.262 | 10.365 | | | | |
| $x0$ | -0.188 | 0.129 | -1.455 | | | | |
| <i>Probability</i> 0 | 0.547 | | | | | | |
| $\sigma0$ | 1.251*** | 0.108 | 11.592 | | | | |
| $\sigma-$ | 2.584*** | 0.207 | 12.477 | | | | |
| likelihood | -5579 | | | | | | |

that the coefficient on current volume under positive state is 5.25, while the coefficient on simultaneous volume under negative state is -4.27 . In our model, when positive information flows in the market, the future price will increase and be associated with higher level of simultaneous volume; when a negative shock generated in the market, the future price will decrease, but that will be associated with higher simultaneous volume as well. When the future price adjusts to its fundamental value, the imbalance in buy and sell orders reduces and more transactions take place. Therefore, the level of volume is higher.

The last group of estimation in Table XIV includes the probability estimates and the conditional standard errors in each state. Five out of seven estimates are significant. The likelihood ratio is also reported in Table XIV.

As we shown above, the empirical results fully support our model prediction.

F. Conclusion

This chapter investigates the relation between trading volume and price movements. We propose a model to explain the dynamics of price and trading volume. Since the model is hard to be analytically solved, Su (2007) develops a trading platform, Mini-Exchange. Traders can generate their limit orders according to our model, and submit them to the Mini-Exchange to trade.

The model's simulation results demonstrate that during the price adjustment process the trading volume is relatively low. And relatively low trading volume predicts a large absolute value in price changes in the future. We implement a mixture normal approach to estimate the relationship between daily return and lags of volume for IBM stock. Empirical results show that trading volume indeed has some predictive power over price movements. The very next thing we want to do is to extend

the data trade by trade by using 5-min data, and see whether the results support our model.

There are many more topics especially topics in liquidity issues we want to study in future research. eg. Thin and Thick Market issues: will the participate rate affects the market behavior and by how? For example, if we observe bid-ask spread becomes smaller, we want to know what the reason is behind. We are confident that from the model we develop we can tell whether the change in bid-ask spread is because more transactions carried on or because information are more accurate. The other discussion we want extend is to understand where comes the liquidity. We can include noise traders or day-traders in our model and see whether those will bring more liquidity to stocks.

CHAPTER IV

DOWNSIDE RISK CONSTRAINTS AND CURRENCY HEDGING IN
INTERNATIONAL PORTFOLIOS: THE ASIAN CRISIS

A. Introduction

International diversification of portfolios has increased markedly in recent years. The maturing of asset markets in developing countries and the increased interest in international diversification among investors worldwide has led to a globalization of investment in asset markets.

This increased internationalization of investment portfolios has led to an increased interest in hedging currency risk. There are many ways to consider hedging currency risk. One approach is to look at minimum variance hedge ratios. Another approach is to look at universal hedging, and yet another is a mean-variance utility approach.

In this paper we consider a joint portfolio choice -hedging decisions, along the lines of Glen and Jorion (1993). We further look at hedging decisions from the unique perspective of an investor with a downside risk constraint, also known as a shortfall constraint.

A shortfall constraint reflects the desire of an investor to limit downside risk, the risk of a significant negative return on a portfolio. Such an investor might also be called a safety-first investor. (See, e.g., Roy (1952) and Telser (1956).) This investor seeks to control downside risk by placing a probabilistic constraint on the maximum loss. Satisfying this constraint, the investor seeks to maximize returns. Alternatively, this can be explained by an investor maximizing returns subject to a Value at Risk constraint.

Our investor wants to minimize the chance of a large negative return, a return

that would reduce his portfolio value below some specified value. We think of such an investor, who we will call a safety-first investor, as motivated by thresholds such as bankruptcy, violation of a legal or regulatory requirement, subsistence levels of wealth or other threshold levels of wealth (such as required wealth for given levels of retirement income), and certain fiduciary responsibilities¹.

We use the statistical theory of extremes (or extreme value theory) to better characterize the risk of extreme returns, and to estimate the probabilities of extreme events with greater accuracy than with the Chebyshev bound. Applications of short-fall constraints or safety first constraints based on assumed normality of returns can yield incorrect conclusions when returns are characterized by excess kurtosis or fat-tailed distributions. We use extreme value theory to model and estimate the tail distribution of returns. Thus our work is related to work by Jansen and Koedijk (1998), Jansen Koedijk and de Vries (2000), and Susmel (2001), who use extreme value theory to model tail probabilities. Our work is also related to Bao et al (2006), who study alternative value-at-risk methodologies for forecasting performance over the Asian Crisis. We add portfolio calculations and a focus on returns to Bao et al, and a dynamic portfolio recalculations and a focus on the Asian Crisis to work by Susmel and others.

To preview our results, we find that in the presence of currency risk and considering the joint decision on portfolio composition and hedge ratios, the portfolio mix is at least as important as the hedging decision in determining portfolio risk.

We apply our model to a hypothetical U.S. investor who invests in a portfolio

¹The safety-first criterion was developed by Roy (1952) at the time Markowitz (1952) was developing mean-variance analysis. Telser (1956) was also influential in the early safety-first literature. Safety-first is related to mean variance and under some conditions the two criteria will yield the same portfolios, see e.g. Levy and Sarnat (1972). These conditions generally require that investors be mean-variance optimizers or that returns are Gaussian.

consisting of a U.S. equity index and an equity index from one of three rapidly developing Asian economies. Moreover, we choose a sample period and countries that includes the Asian crisis of 1997, so that large currency movements and large equity price movements were relatively common. This is a period during which extreme realizations were common, and this period serves as a stress-test of the ability of safety-first optimization.

B. Safety First Portfolio Choice: Theory

We consider a one-period model. At the beginning of the period investors can invest in a risky domestic asset, a risky foreign asset, and a safe domestic asset. We do not allow short positions except that the investor may borrow or lend in terms of the safe domestic asset at the risk-free rate.

The safety-first investor wants to first make sure that his portfolio satisfies a risk constraint, and then maximize expected returns among the set of portfolios satisfying the risk constraint. The problem can be stated in an equivalent form as maximizing expected returns subject to a particular shortfall constraint or downside risk constraint. We present the problem in terms of a shortfall constraint.

At the beginning of the period the investor faces market prices of asset j in local currency terms, denoted $P_{j,t}$. The spot exchange rate in the reference currency (e.g. USD per Korean Won) is $S_{j,t}$. Initial wealth is W_t , and b_t is the amount of borrowing (or lending; negative amounts indicate lending). $Q_{j,t}$ is the amount of risky asset j held by the investor at the beginning of time t . Finally, R_t^{rf} is the risk free gross rate of return, known at time t .

Initial wealth is allocated among assets so that at the beginning of period t :

$$\sum_j Q_{j,t} P_{j,t} S_{j,t} = W_t + b_t. \quad (4.1)$$

We allow investors to hedge currency risk. We consider currency hedging as occurring in the forward market. A U.S. investor hedges some portion of a foreign currency-denominated equity position by shorting the foreign currency, and lending U.S. dollars. This is modelled by including a term that measures the gain in the forward market, $(S_{j,t+1} - F_{j,t})$, where $F_{j,t}$ is the forward foreign exchange rate in dollars per unit of country- j foreign currency at time t . Obviously the forward rate is known at time t , and the future spot rate is uncertain.

Investor expected wealth at the end of the period, i.e. at time $t + 1$, can be written as:

$$\mu_t = E_t \{ \sum_j Q_{j,t} P_{j,t+1} S_{j,t+1} - \sum_j Q_{j,t} h_j P_{j,t} (S_{j,t+1} - F_{j,t}) - b_t R_t^{rf} \} \quad (4.2)$$

Here the first summation on the right hand side is the U.S. dollar value of risky assets, both foreign and domestic, at time $t+1$. The second summation is the value of hedging transactions, where h_j represents the hedge ratio, the amount of the equity position that is hedged. The final term represents the reduction in final period wealth from repaying loans at the risk free rate.

Our safety-first investor will maximize expected final-period wealth as given in equation (2), subject to the wealth constraint in equation (1) and to a downside risk or shortfall constraint. The shortfall constraint can be stated in a number of different ways. Here we choose to write it in a way that emphasizes the safety-first nature of the portfolio problem we examine. The safety-first constraint is a constraint that requires chosen portfolios to satisfy

$$Probability\{sum_j Q_{j,t} P_{j,t+1} S_{j,t+1} - sum_j Q_{j,t} h_j P_{j,t} (S_{j,t+1} - F_{j,t}) - b_t R_t^{rf} \leq s\} \leq \delta. \quad (4.3)$$

Here the safety first investor has two preference parameters, the disaster level of wealth, s , and the maximal acceptable probability of this disaster, δ .

We restate the problem in terms of returns, to state the problem as maximizing expected returns subject to a shortfall constraint. The gross returns on the risky portfolio between period t and $t + 1$, R_{t+1} , is a function of portfolio proportions and hedging ratios determined at time t . We can write these gross returns as:

$$R_{t+1} = \left(\frac{sum_j Q_{j,t} P_{j,t+1} S_{j,t+1} - sum_j Q_{j,t} h_j P_{j,t} (S_{j,t+1} - F_{j,t})}{sum_j Q_{j,t} P_{j,t} S_{j,t}} \right) \quad (4.4)$$

or

$$R_{t+1} = \sum_j \gamma_{j,t} R_{j,t+1}^P R_{j,t+1}^S - \sum_j \gamma_{j,t} h_{j,t} R_{j,t+1}^S + \sum_j \gamma_{j,t} h_{j,t} (F_{j,t} / S_{j,t}) \quad (4.5)$$

where γ indicates the proportion of the portfolio in asset j , or

$$R_{t+1} = \sum_j \gamma_{j,t} R_{j,t+1}^P R_{j,t+1}^S - \sum_j \gamma_{j,t} h_{j,t} [R_{j,t+1}^S - (F_{j,t} / S_{j,t})] \quad (4.6)$$

Note the third term on the right hand side of equation (6) is known at time t , so the unknown terms are the local currency return (specified as the first term on the right hand side of equation (6)) and the return on the spot exchange rate (specified as the second term on the right hand side of equation (6)).

The shortfall constraint, written in terms of returns, is

$$Probability(R_{t+1} \leq R_t^{rf} + \frac{W_t R^{Min} - W_t R_t^{rf}}{W_t + b_t}) \leq \delta \quad (4.7)$$

Here we write the safety-first disaster level of wealth, s , as a minimum gross

return on initial wealth, or R^{Min} times W . Presumably the minimum gross return is below unity, capturing a negative net return.

We define the quantile value $q_\delta(R)$ such that there is a δ percent chance of returns less than or equal to this value. We can define the quantile value implicitly as

$$Probability(R_{t+1} \leq q_\delta(R)) = \delta \quad (4.8)$$

Then the safety first criterion is violated whenever

$$q_\delta(R) < R_t^{rf} + \frac{W_t R^{Min} - W_t R_t^{rf}}{W_t + b_t} \quad (4.9)$$

The quantile $q_\delta(R)$ is also the value at risk, or VaR, with probability δ for the given portfolios return on risky assets, R . Thus the shortfall constraint in this problem is equivalently a Value-at-Risk constraint.

C. Characterizing A Safety-First Investor

Our safety first investor will exhibit risk aversion if the critical wealth level s is smaller than his secure final wealth, his initial wealth invested at the risk-free rate R^{rf} , or $W \cdot R^{rf}$.

A risk averse safety-first investor will decline a fair risk that violates the safety first criterion in favor of pure lending at the risk free rate.

The risk averse safety-first investor will buy some part of a divisible favorable risk, however, and the amount will be such as to satisfy the equation (8) above as an equality. That is,

$$W_t + b_t = \frac{W_t R^{Min} - W_t R_t^{rf}}{q_\delta(R) - R^{rf}} \quad (4.10)$$

If some favorable risks are available, the portfolio problem can be rewritten as:

$$\max_{\gamma_j, h_j, b} \mu = W_t R^{rf} + (W_t + b_t) E_t(R_{t+1} - R^{rf})$$

Here the maximization is carried out among those portfolios that satisfy the safety first criterion. But, for these portfolios, problem (10) reduces to

$$\max_{\gamma_j, h_j, b} \mu = W_t R^{rf} - (W_t R^{Min} - W_t R^{rf}) E_t(R_{t+1} - R^{rf}) / (R^{rf} - q_\delta(R)) \quad (4.11)$$

This gives a portfolio separation result. The risk averse safety first investor first maximizes the ratio of the risk premium to the return opportunity loss that he is willing to incur with probability δ , or

$$\max_{\gamma_j, h_j} \frac{E_t(R_{t+1} - R^{rf})}{(R^{rf} - q_\delta(R))} \quad (4.12)$$

Second, the investor can pick the scale of the risky part of his portfolio, i.e., the borrowing (or lending) amount b_t .

A major practical problem with safety-first portfolio selection is determining VaR. Given the risk level δ , the portfolio distribution determines $q_\delta(R)$. Some researchers have assumed normality, but the statistical tests overwhelmingly reject normality as a characterization of the distribution of returns. In fact, the actual distribution of returns is not known, but it is relatively well established that return distributions have fat tails. Roy and others have used the Cheybshev bound to estimate VaR, but this is a poor approximation and a very loose upper bound that in practice can be completely uninformative (such as giving an upper bound of one.) We use extreme value theory to provide a better approximation, a tighter estimate of the VaR. This allows us to measure VaR for small δ without making strong distributional assumptions. On the other hand, extreme value theory is of little or no use for large δ .

D. Extreme Value Theory

Consider a stationary sequence X_1, X_2, \dots of i.i.d. random variables with distribution function $F(\cdot)$. The probability that the maximum of the first n random variables, $M_n = \max(x_1, \dots, x_n)$, is below a certain value x is given by $P(M_n < x) = F^n(x)$.

Extreme value theory studies the limiting distribution of the order statistic M_n . In particular, the distribution function $F^n(x)$ of M_n converges, when suitably normalized and for large n , to a limiting distribution $G(x)$, where $G(x)$ is one of only three asymptotic types.

Analogies can be drawn to the central limit theorem. Various parent distributions lead to similar distribution of a sample statistic calculated from the tails of the sample, at least in the limit.

There are three possible limiting distributions instead of the single limiting distribution in the central limit theorem, we can narrow these down by using additional information. One of the three types of limiting distributions holds for parent distributions with finite support. We exclude this type on a priori grounds when looking at stock returns².

Of the remaining two, one is characterized by the existence of all moments (such as, e.g., a Normal parent distribution), and the other by the absence of higher moments (such as, e.g., the Students t distribution or the Stable distribution).

Because stock returns are fat-tailed relative to the normal distribution, we consider the limiting distribution $G(x)$ which is characterized by a lack of some higher

²With returns measured as log differences of index levels, both negative and positive returns are in principle unbounded.

moments³. This limiting distribution, for a suitably normalized x , is:

$$G(x) = \begin{cases} 0 & \text{if } x < 0 \\ \exp(-x)^{-\alpha} & \text{if } x > 0 \end{cases} \quad (4.13)$$

In equation (12) the parameter α is known as the tail index, and indicates the rate of decline in the tail of the distribution.

What distributions are included in $G(x)$? Examples would be the Student t with finite degrees of freedom, the stable distribution, and the ARCH process. For the Student t , the tail index α is the degrees of freedom. For the symmetric stable Paretian, the tail index can be interpreted as the characteristic exponent, which falls in the interval $(0,2)$.

A necessary and sufficient condition for $F(x)$ to be in the domain of attraction of $G(x)$ is the regular variation property, i.e. $\lim[1 - F(tx)]/[1 - F(t)] = x^{-\alpha}$ as $t \rightarrow \infty$. Note that the tail of the normal distribution, a thin-tailed distribution with all moments existing, is not characterized by the $G(x)$ given above.

To apply the theory of extremes, we assume that the stock return series can be characterized by the distribution $G(x)$ given above. We estimate α , which provides information on the tail shape of the limiting distribution and also indicates the number of moments that exist. This is interesting in itself, as it provides information on how many moments exist in the distribution of returns without conditioning the answer on the form of the unknown distribution.

More important, the estimate of α allows us to calculate exceedence values, values of x that will only be exceeded with a stated probability. In fact, we can calculate exceedence values for probabilities that are much lower than $1/n$ of our sample! By relying on the limit distribution in the tail, we can sensibly extrapolate

³See, for example, Jansen and de Vries (1991) and Loretan and Phillips (1994).

the tail far outside the sample experience to make statements about the exceedence values corresponding to very small probabilities.

Turning the problem around, we can state a large value x , even one far outside the sample experience, and calculate the probability of its occurrence.

To estimate the tail index α we use Hill's (1975) moment estimator. We first obtain the order statistics $X_{(n)}, X_{(n-1)}, \dots, X_{(1)}$ from our random variables, where $X_{(n)} > X_{(n-1)} > X_{(n-2)}$, etc. The Hill estimator is:

$$\gamma = 1/\alpha = \frac{1}{m} \sum_{i=1}^m [\log(X_{(n+1-i)}/X_{(n-m)})] \quad (4.14)$$

where m is the number of upper order statistics included in the estimator.

The Hill estimator may be biased in small samples, depending on the underlying distribution, but Mason (1982) proves that under some regularity conditions the Hill estimator is consistent for γ . Furthermore, Goldie and Smith (1987) show that $\{(1/\alpha) - (1/\hat{\alpha})\}\sqrt{m}$ is asymptotically normal $N(0, 1/\alpha^2)$ if m increases suitably rapidly as $n \rightarrow \infty$. Danielsson et al (1996) provide Monte Carlo evidence of the size of the bias in estimates of α if m increases less rapidly. An important issue in using the Hill estimator is the choice of order statistics, m . The choice of m should be such that $m(n)$ goes to infinity with n , but m/n remains finite.

We use the estimation methodology Drees and Kaufmann (1998). Theirs is a sequential procedure for estimating m that is consistent under fairly general conditions. They start with the idea that the Hill estimator is biased when used to estimate the tail index of a non-Pareto distribution, and this bias leads to fluctuations in the estimated tail index as the tail size is increased, i.e. as the number of order statistics used in the Hill estimator, m , is increased. To estimate m using a stopping rule that compares these fluctuations in the estimated tail index with a pre-defined threshold, and the value of m is set where the threshold is exceeded. Drees and Kaufmann show

that their resulting adaptive Hill estimator is asymptotically as efficient as the Hill estimator based on the (unknown) optimal m^4 .

After calculating our estimate of the tail index, α , we can estimate the quantiles q_p . These are the exceedence quantiles, estimates of values that will only be exceeded with probability p , where p is on the order of $1/n$ or smaller.

Based on a corollary to Dekkers et al. (1989) derived in the appendix of de Haan et al. (1994), we have:

$$\hat{q}_p = X_{n-m} \left(\frac{m}{pn} \right)^{1/\alpha} \quad (4.15)$$

This provides us with a method to estimate the VaR.

E. Preliminary Data Analysis

We have weekly observations from four countries on portfolio returns, spot exchange rates, and interest rates. One country is the United States, which we take as the home country of our hypothetical investor. The other three countries are Indonesia, Korea, and Thailand. We chose these countries because they were countries impacted to a large extent by the Asian Crisis that began in July 1997.

Our data begins at different times for these three countries. For Indonesia we have 825 observations beginning the week of April 25, 1990. For Korea, 653 observations beginning the week of August 13, 1993. For Thailand 735 observations beginning the week of January 15, 1992. All series end the week of February 8, 2006.

The data is largely from Datastream. Portfolio returns are local currency returns on equity portfolios (with dividends reinvested) as calculated by Datastream. The

⁴We use an initial consistent estimate of the tail index based on using $2\sqrt{n}$ order statistics in the Hill estimator. The stopping rule is a random threshold $2.5\gamma n^{.35}$ where γ is the initial consistent estimate of the tail index. Tuning parameters are $\lambda = 0.6$ and $\zeta = 0.7$. We take $\rho = 1$.

interest rates are a variety of series as available from Datastream. For the U.S., the interest rate used for the forward premium is the one-week euro-dollar rate, from Datastream. For the risk free rate we use the U.S. 3-month Treasury Bill rate. For Indonesia we use the interbank call rate from Datastream. For Korea we use the overnight call rate from Datastream. (A 10-day or 1-week rate was not available.) For Thailand we use the 10-day money market rate from Datastream. Finally, the spot exchange rates are from the St. Louis Federal Reserve Banks FRED databank.

Table XV presents summary statistics for data in our sample. The first third of the table presents summary statistics on the local currency return on equity. The U.S. has the largest mean return and the smallest standard deviation, as well as the lowest maximum return and the largest minimum return.

The middle third of the table provides information on spot exchange rates (again in U.S. dollars per unit foreign currency). All the Asian countries have negative mean values. The standard deviation is highest for Indonesia then Korea, then Thailand. The maximum and minimum returns tell a similar story.

The bottom third of Table XV report U.S. dollar returns on equity in the Asian countries. These are lower than the local currency returns because of the spot exchange rate returns. The variance of the U.S. dollar returns is above the variance of local currency returns. The maximum and minimum returns are large, indicative of the volatility, and Indonesia has the highest volatility and, in terms of extremes, the maximum return in Indonesia exceeds 40%, and the minimum return is below -40% .

F. Our Safety First Application

We begin by choosing parameters for the safety first investor preferences. We set $R^{min} = .8$, or a minimum net return of -20% , and we set $\delta = 1/520$. These choices

Table XV. Summary Statistics: Entire Sample

| | Mean | Std Dev | Max. | Min. | Kurtosis | Sample |
|--|----------|---------|---------|----------|----------|--------|
| Local currency returns on equity, continuous compounding | | | | | | |
| Indonesia | 0.0011 | 0.0427 | 0.2276 | -0.20391 | 4.34 | 825 |
| Korea | 0.00205 | 0.04536 | 0.25246 | -0.22538 | 3.37 | 653 |
| Thailand | 0.00127 | 0.04472 | 0.20694 | -0.16522 | 2.14 | 735 |
| U.S. | 0.00227 | 0.02136 | 0.1002 | -0.09143 | 2.07 | 927 |
| Spot exchange rate returns (US dollar per unit local currency), continuous compounding | | | | | | |
| Indonesia | -0.00197 | 0.03921 | 0.35525 | -0.57721 | 76.45 | 825 |
| Korea | -0.00027 | 0.02046 | 0.13451 | -0.33276 | 113.87 | 653 |
| Thailand | -0.0006 | 0.0142 | 0.0989 | -0.13649 | 26.36 | 735 |
| Dollar returns on equity, continuous compounding | | | | | | |
| Indonesia | -0.00087 | 0.06231 | 0.43991 | -0.47549 | 15.66 | 825 |
| Korea | 0.00178 | 0.05333 | 0.27275 | -0.3672 | 6.69 | 653 |
| Thailand | 0.00067 | 0.04879 | 0.27667 | -0.19785 | 2.71 | 735 |

imply that the safety first investor wants the probability of a 20% weekly decline in wealth to be once in 10 years. We choose these values to model an investor who wants a large negative return on his portfolio to be rare. We then examine how this safety-first investor would have fared during the Asian Crisis.

Our simulated real time safety-first portfolio investor proceeds as follows. We are considering a U.S. investor allocating his portfolio of risky assets between U.S. equity and the equity of one of the Asian countries in our study. For each of the three Asian countries, we start with data from the beginning of our samples to the end of June 1997. We say that the span of this data is from 1 to n , where n varies by country. For this data, we estimate the VaR (i.e. $q_\delta(R)$) for different portfolios of U.S. equity and foreign equity and different hedge ratios. We consider portfolio proportions $\gamma \in [0, 1]$.

That is, we do not allow short sales. We consider hedge ratios $h \in [0, 1]$.

For each possible portfolio—the complete set of combinations of γ and h —we estimate expected returns. This consists of the three components in equation (6). Expected returns on spot exchange rates are assumed to be equal to the forward premium on foreign exchange. Expected returns on equity for both the U.S. and the relevant Asian country are assumed to be a 52-week moving average of lagged returns.

Given estimated portfolio returns, we choose the optimal portfolio—the optimal values of γ and h —according to equation (12). Then we calculate the optimal scale of the portfolio, the optimal borrowing amount, b , from equation (10).

Once we have decided on the portfolio for the first period— γ , h , and b —we observe the actual return in period t and calculate the portfolio's return based on the optimal values of γ , h , and b calculated on data from 1 to n .

We then advance the sample to include data from 1 to $(n + 1)$, and proceed to calculate the optimal portfolio for period $(n + 1)$. We proceed in this fashion, iteratively, through to the final observation in February 2006.

G. Results

Figure 16 to 18 provide graphs of equity returns and currency values for Indonesia. Figure 16 shows the dollar return on Indonesian equity as compared to U.S. equity over a period from 1990 through 2006. Indonesian equity is more volatile than U.S. equity over the entire sample, and the large increase in volatility of Indonesian equity returns starting in mid-1997 is apparent. The average weekly return on U.S. equity is substantially higher than the (negative) average weekly return in dollars of Indonesian equity, and the volatility of Indonesian dollar equity returns is much higher than for the U.S.

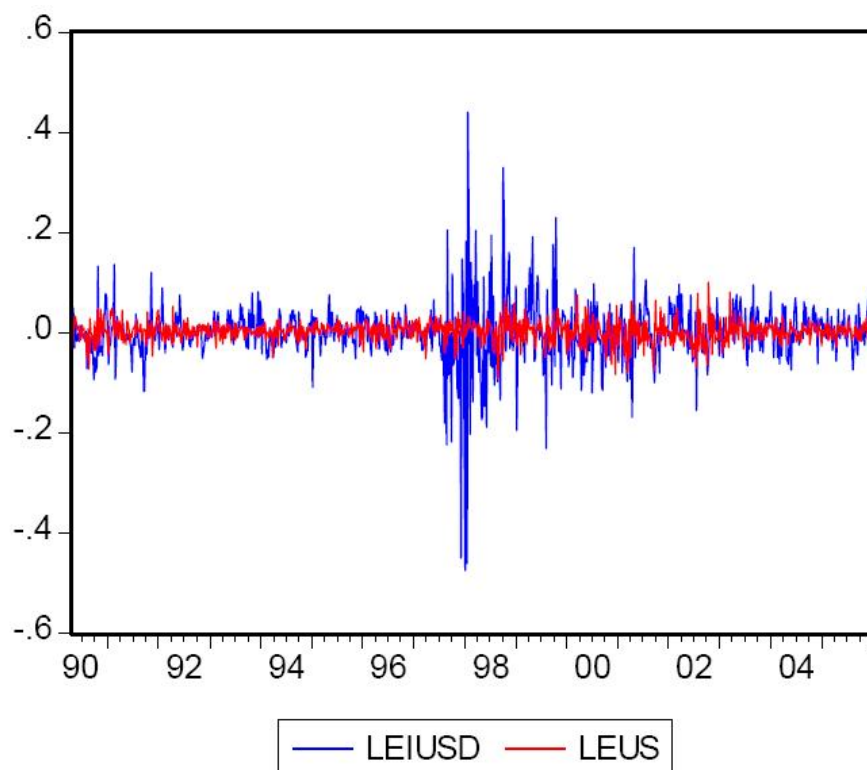


Fig. 16. Dollar Return on U.S. and Indonesian Equity

Figure 17 shows the local currency returns, and while the differences in volatility between Indonesian and U.S. returns are somewhat attenuated it is still clear that Indonesian equity has a higher volatility and that this volatility increased substantially in mid-1997. Figure 18 shows the behavior of the U.S. dollar-Indonesian Rupiah exchange rate, including the great depreciation in 1997 during the Asian crisis. This graph includes a line at -20% as a point of reference.

Figure 19 shows the evolution of our hypothetical safety-first portfolio from late June 1997 onward, including both the scale of the portfolio as indicated by borrowing, and portfolio holdings including the percent invested in Indonesia and the hedge ratio on the Indonesian investment. Borrowing is shown in the bottom graph, and is about

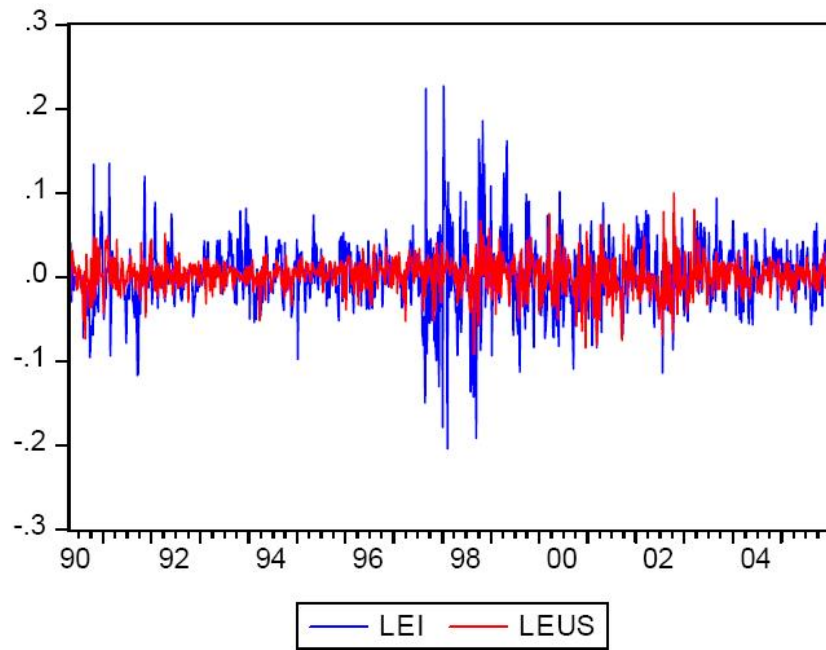


Fig. 17. Local Currency Return on U.S. and Indonesian

\$1.75 per \$1 of initial wealth until mid-2000, then falls to about -\$0.5 per \$1 of initial wealth until 2003, after which borrowing rises to almost \$1 per \$1 of initial wealth through the end of the sample. There is of course some volatility and changes in borrowing apart from these large movements. The top graph in Figure 19 plots portfolio holdings and hedge ratios. Our U.S. investor holds little Indonesian equity until mid-2000, and what is held is heavily hedged. Starting in mid-2000 the portfolio is invested nearly completely in Indonesia, and unhedged from mid-2000 through mid-2001, after which the hedge ratio gets very large. Then near the beginning of 2003 the proportion of Indonesian equity is about 40%, and hedging is nearly complete, through the end of the sample.

Figure 20 compares the portfolio return in dollars of a 100% U.S. equity portfolio, a 100% Indonesian equity portfolio, as well as the dollar return on the risky portion

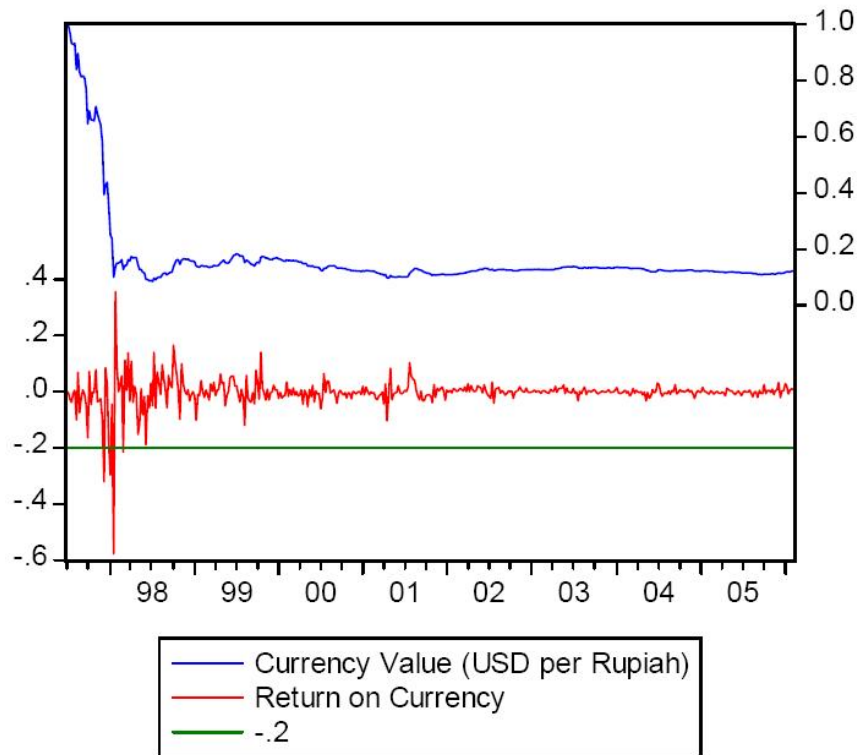


Fig. 18. Devaluation of Indonesian Currency

of the safety-first portfolio, and the dollar return on the complete safety-first portfolio including risk-free investments. Note that the dollar return on (unhedged) Indonesian equity is miserable, with a portfolio value falling to about 10% of its initial value. The value of a U.S. equity investment rises to about 150% of its initial value by the end of our sample. Meanwhile the return on the safety first investment is nearing 250% by the end of our sample. Thus our hypothetical safety-first investor manages to earn a return greater than an investor fully invested in U.S. equity or fully invested in Indonesia. Finally, borrowing adds to the final portfolio value. The increase in value of the risky part of the safety-first portfolio is near 200%, and this falls short of the 250% increase in the full safety-first portfolio that includes the return on borrowing

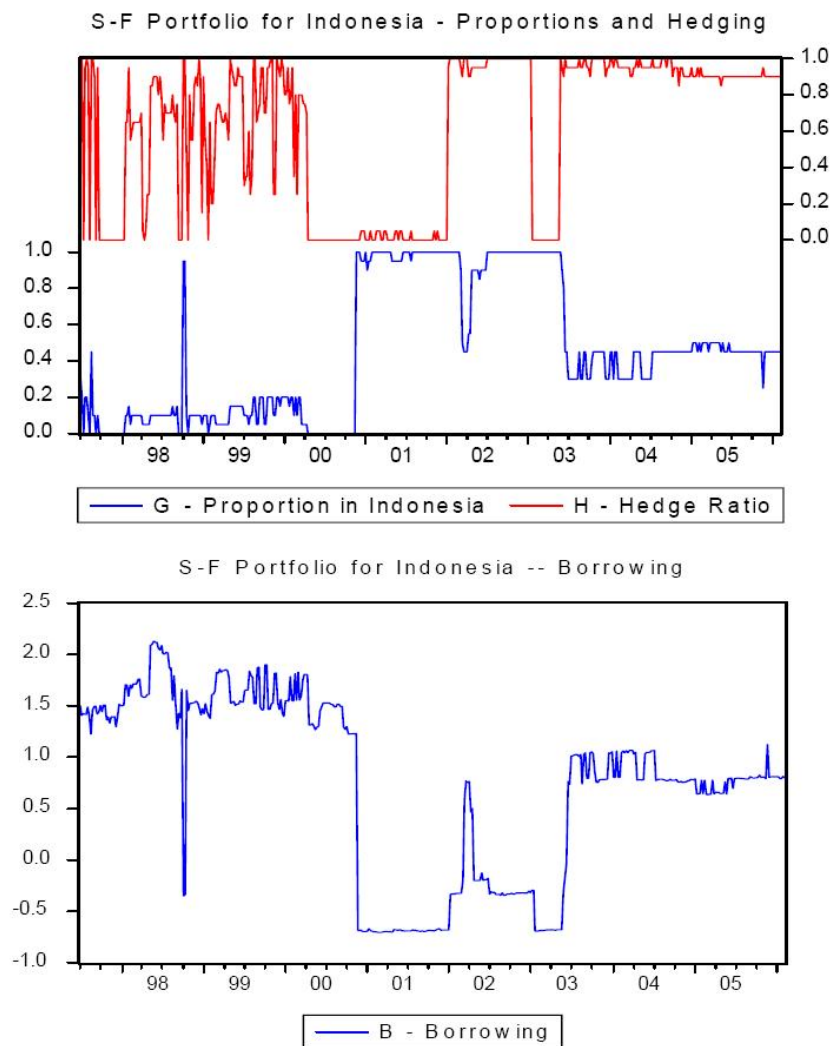


Fig. 19. S-F Portfolio for Indonesia

or lending activities as well as on the risky part of the portfolio. So over this period leverage works to increase the value of the safety-first portfolio for an investor in Indonesian equity.

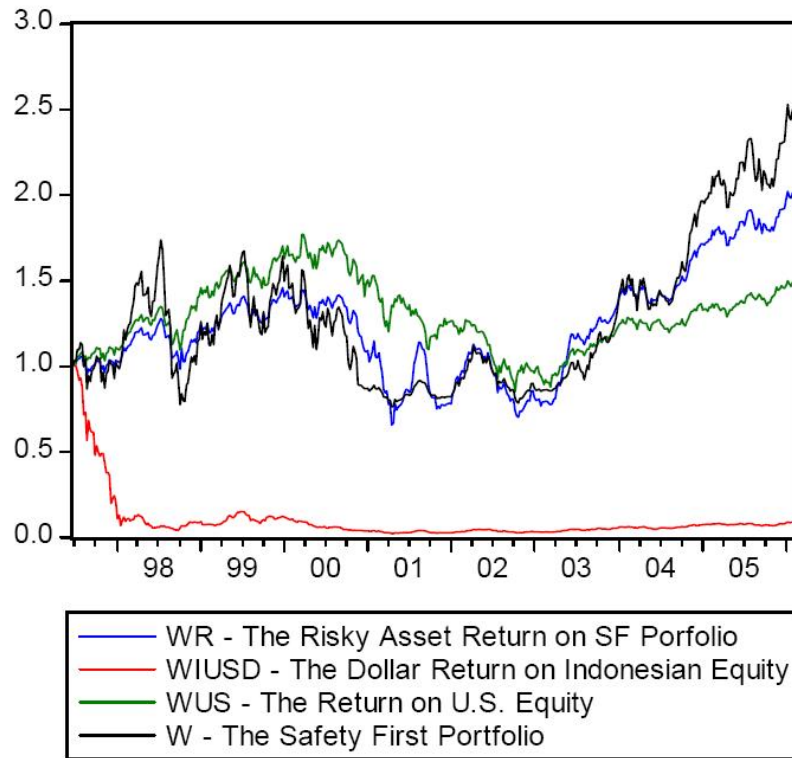


Fig. 20. Performance of S-F Portfolio Versus Country-Specific Index

Figure 20 also shows that the safety-first portfolio was not immune to downturns, but it was fairly immune to the early stages of the Asian crisis, largely because the investor was only slightly invested in Indonesia, and that investment was heavily hedged, at the time of the Asian crisis. The decline in the safety-first portfolio in mid-1998 was in the aftermath of the Asian crisis. Even this decline was due in part to a heavily leveraged portfolio and a decline in the U.S. market values. The decline in U.S. equity returns in 2000 also had a large impact, and led our hypothetical safety

first investor to invest in Indonesian stocks over this period and also to lend about half of his portfolio at the risk free rate. Finally, the steady increase in value of the safety-first portfolio from late 2002 is due to the rising value of both U.S. and (hedged) Indonesian equity coupled with a fairly high leverage rate.

Figure 21 to 23 provide graphs of equity returns and currency values for Korea. Figure 21 shows the dollar return on Korean and U.S. equity over the period 1993 to 2006. Korean equity is more volatile than U.S. equity over this entire period, and there is a clear large increase in volatility of Korean equity returns starting in mid-1997, the Asian Crisis period. The average weekly return on U.S. equity is higher than for Korean equity, but much closer than in the case of Indonesia. The volatility of Korean dollar equity returns is much higher than for the U.S.

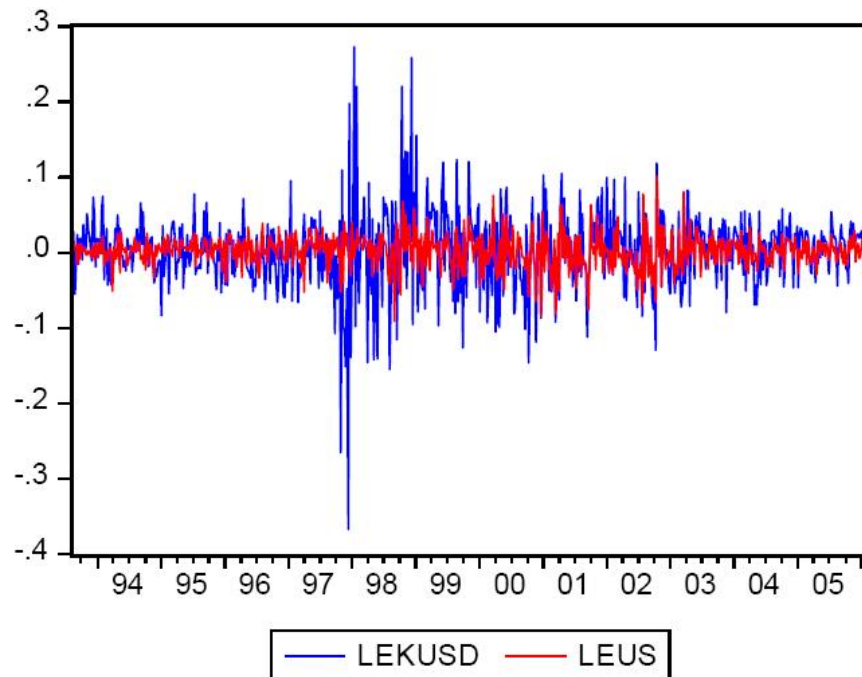


Fig. 21. Dollar Return on U.S.(LEUS) and Korean (LEKUSD) Equity

Figure 22 shows the local currency returns, and here the differences in volatility between Korean and U.S. returns is hard to distinguish from Figure 21. However, Korean volatility is somewhat diminished while still much higher than the volatility of U.S. returns. Interestingly, the average rate of return in local currency of Korean equity slightly exceeds that of U.S. equity over this period.

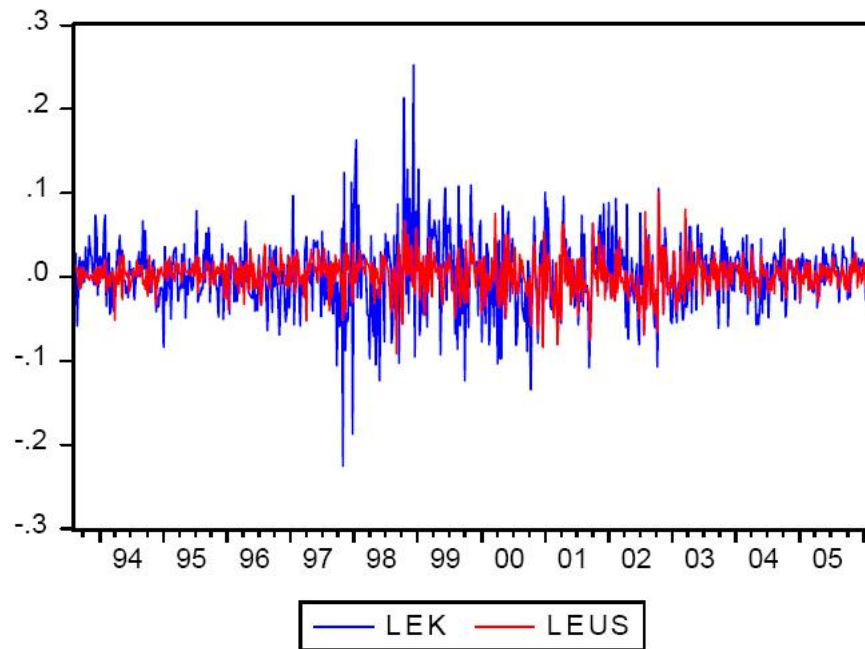


Fig. 22. Local Currency Return on U.S.(LEUS) and Korean (LEK) Equity

Figure 23 shows the behavior of the U.S. dollar-Korean Won exchange rate, including the depreciation in 1997 during the Asian crisis. The Won initially fell to about 60% of its pre-crisis value but then recovered over the next year to near 70% of its pre-crisis value, and by early 2006 was at 80% of its pre-crisis value.

Figure 24 shows the evolution of our hypothetical safety-first portfolio from late June 1997 onward, including both the scale of the portfolio as indicated by borrowing, and portfolio holdings including the percent invested in Indonesia and the hedge

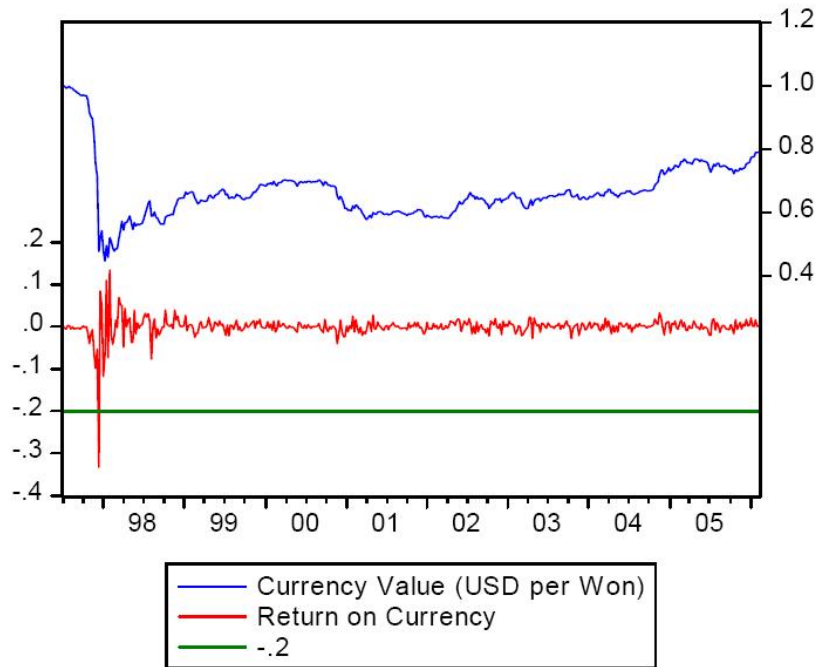


Fig. 23. Devaluation of Korea Won

ratio on the Indonesian investment. Borrowing is shown in the bottom graph, and is initially above \$3 per \$1 of initial wealth, a ratio that declines fairly steadily to about \$0.5 per \$1 of initial wealth in mid-1999 and then drops abruptly to a value somewhat below zero. There is some volatility and a jump in early 2000 to a borrowing amount near \$1 per \$1 of initial wealth until late 2000, at which point the borrowing ratio again falls to a value below zero. It moves between about -\$0.5 and zero until mid-2003, when borrowing again approaches \$1 per \$1 of wealth, a value that plateaus but then diminishes to near zero again near the end of 2005.

The top graph in Figure 24 plots portfolio holdings and hedge ratios. Our U.S. investor holds little Korean equity until mid-1999, and what is held is heavily hedged. Starting in 1999 the portfolio is invested nearly completely in Korea but with periods

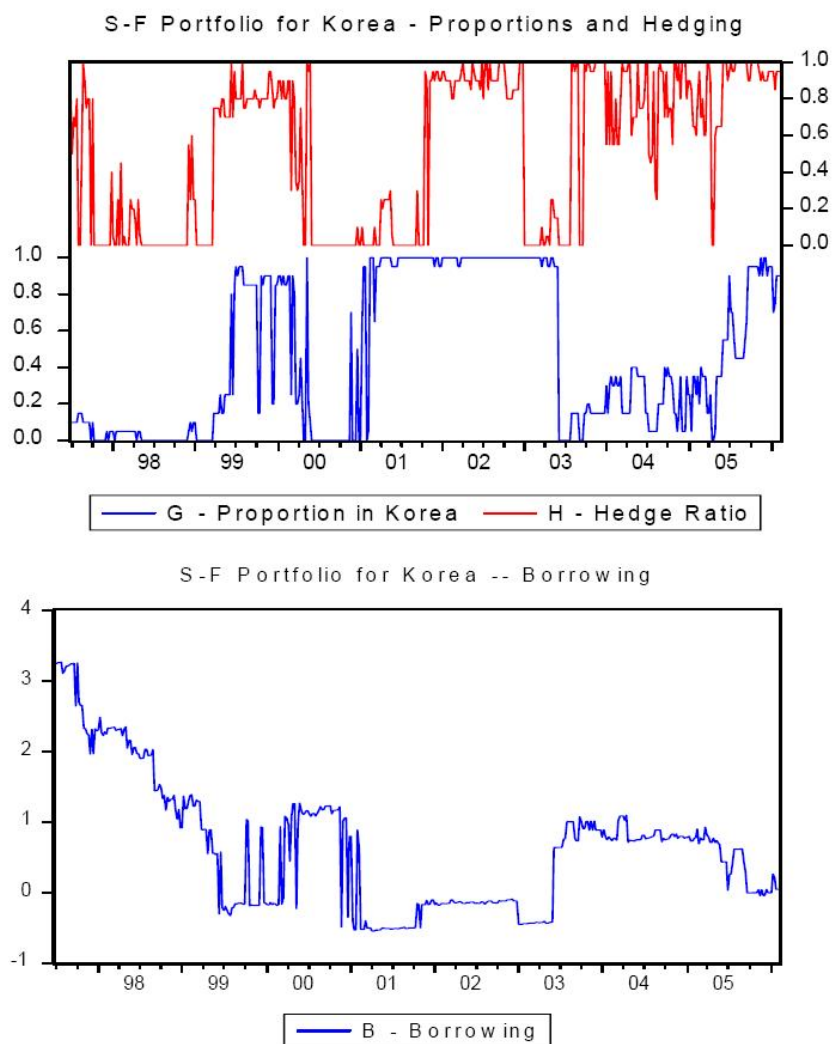


Fig. 24. S-F Portfolio for Korea

of volatility in the holdings, and Korean holdings were heavily hedged, until early 2000. In early 2000 holdings of Korean equity fell to near zero and stayed there until early 20001, at which point Korean equity was held almost completely, and unhedged, until late 2001, at which point Korean equity was hedged almost totally. The hedge ratio fell to near zero in early 2003, and Korean equity holdings fell to zero in mid-2003, and then fluctuated between zero and about 35% until mid-2005, at which point holding of Korean equity again jumped to be the majority of the portfolio.

Figure 25 compares the portfolio return in dollars of a 100% U.S. equity portfolio, a 100% Korea equity portfolio, as well as the dollar return on the risky portion of the safety-first portfolio, and the dollar return on the complete safety-first portfolio including risk-free investments. Note that the dollar return on (unhedged) Korean equity is low C essentially zero over the entire period from July 1997 to February 2006 C but much better than the comparable dollar return on (unhedged) Indonesian equity over this period (see Figure 20). The safety-first portfolio increases to 180% of its initial July 1997 value, while the U.S. equity portfolio increases to 140%. Thus the safety-first portfolio outperforms a buy-and-hold strategy for U.S. equity. At the same time, the safety-first portfolio earns some large losses such as in 2000, when it declines hugely in response to declining U.S. and Korean equity values.

Figure 26 to 28 graph equity returns and currency values for Thailand, the third country hard hit by the Asian Crisis. In Figure 26 we see the familiar graph showing how much larger is the volatility of dollar returns on Thai equity as compared to U.S. equity, and how that volatility increased in the period beginning July 1997. The dollar returns on Thai equity are not only more volatile but are substantially lower on average than the returns on U.S. equity.

Figure 27 shows a similar graph for local currency returns, and as was the case for Korea it is somewhat difficult to see any attenuation of volatility, although the

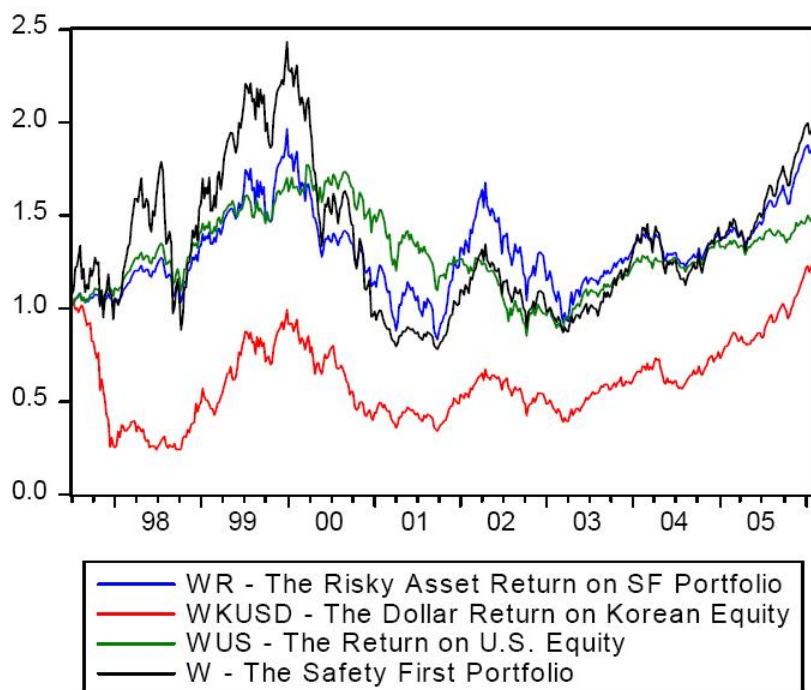


Fig. 25. Performance of S-F Portfolio Versus Country-Specific Index

volatility is somewhat lower for local currency returns in Thailand. The mean return in local currency for Thai equity remains lower than the mean U.S. return, but the difference is much less than it is when we look at dollar returns.

Figure 28 graphs the currency value of the Thai Baht. The large declineCroughly 50%Cthat occurs beginning in July 1997 is obvious, as is the subsequent partial recovery in early 1998. Still, by February 2006 the Baht is still only at 60% of its value at the end of June 1997.

The above graphs indicate that Thailand in many ways falls between Korea and Indonesia in terms of severity of the Asian Crisis, with Indonesia hit hardest. However, it is clear that all three nations suffered severe currency depreciations and severe stock market reactions during the period that has been labeled the Asian Crisis.

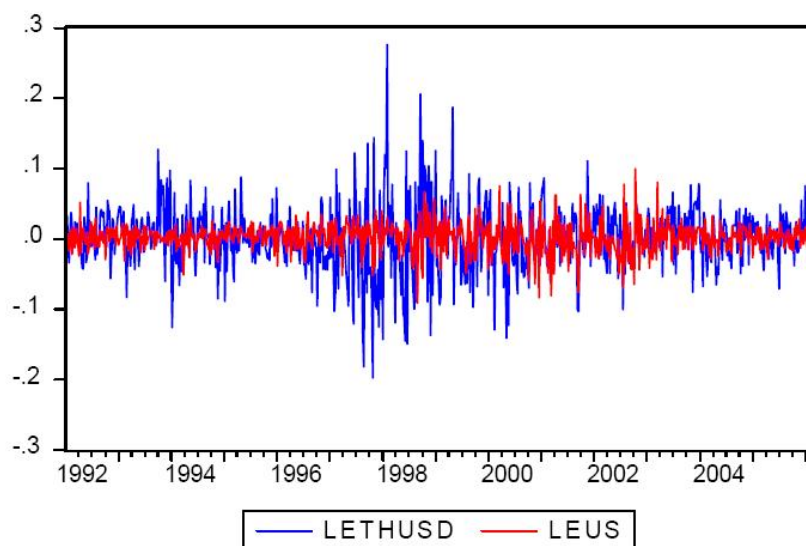


Fig. 26. Dollar Return on U.S. (LEUS) and Thai (LETHUSD) Equity

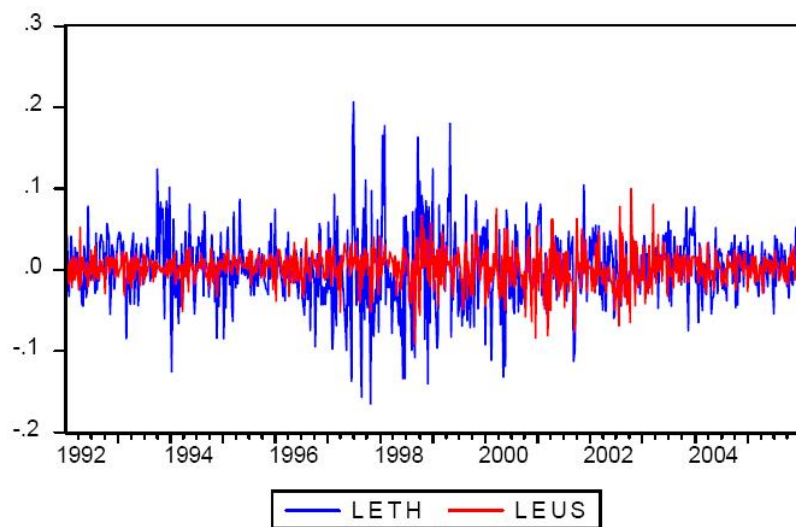


Fig. 27. Local Currency Return on U.S. (LEUS) and Thai (LETH) Equity

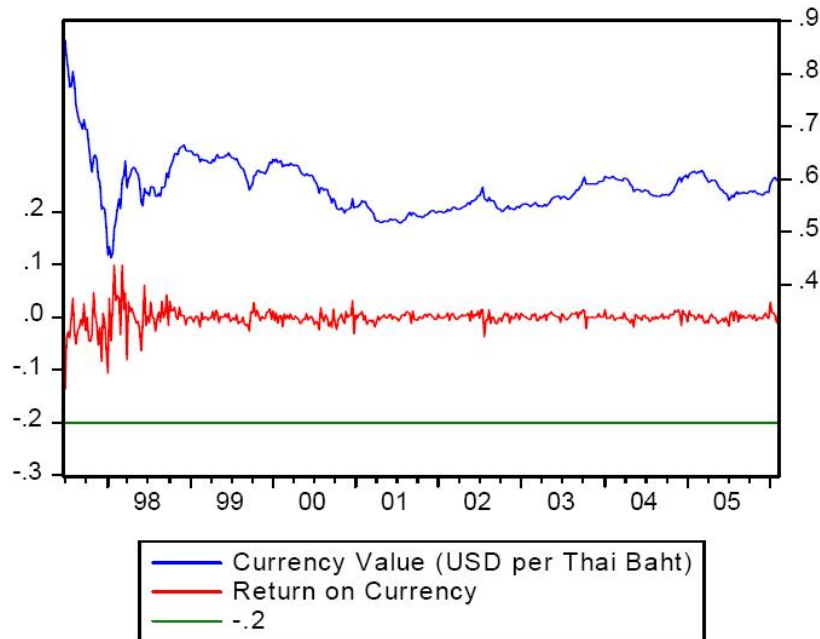


Fig. 28. Devaluation of Thai Baht

Figure 29 shows the evolution of our hypothetical safety-first portfolio from late June 1997 onward. Our safety-first investor would hold almost no Thai stock from July 1997 until early in 2001, and then would hold almost all Thai stock until mid-2003, when holdings would be reduced to about 20%. When Thai stock is held in the 2001-2003 period it is partially hedged, with a volatile hedge ratio that seems to vary around about 40%. After mid-2003 the hedge ratio rises and, while still volatile, is often around 80% or so.

Figure 29 also shows the leverage on the safety-first portfolio, with almost \$2 borrowed per \$1 of portfolio wealth in 1997, declining sharply in mid-1998 to about \$1.2 per \$1 of wealth, varying around this ratio until early 2001 and then dropping to almost -\$0.4, or lending almost \$0.4 of each \$1 of portfolio wealth. Borrowing varied between -\$0.4 and \$0 until mid-2003, when it increased sharply to the \$1.2 range,

where it remained through February 2006.

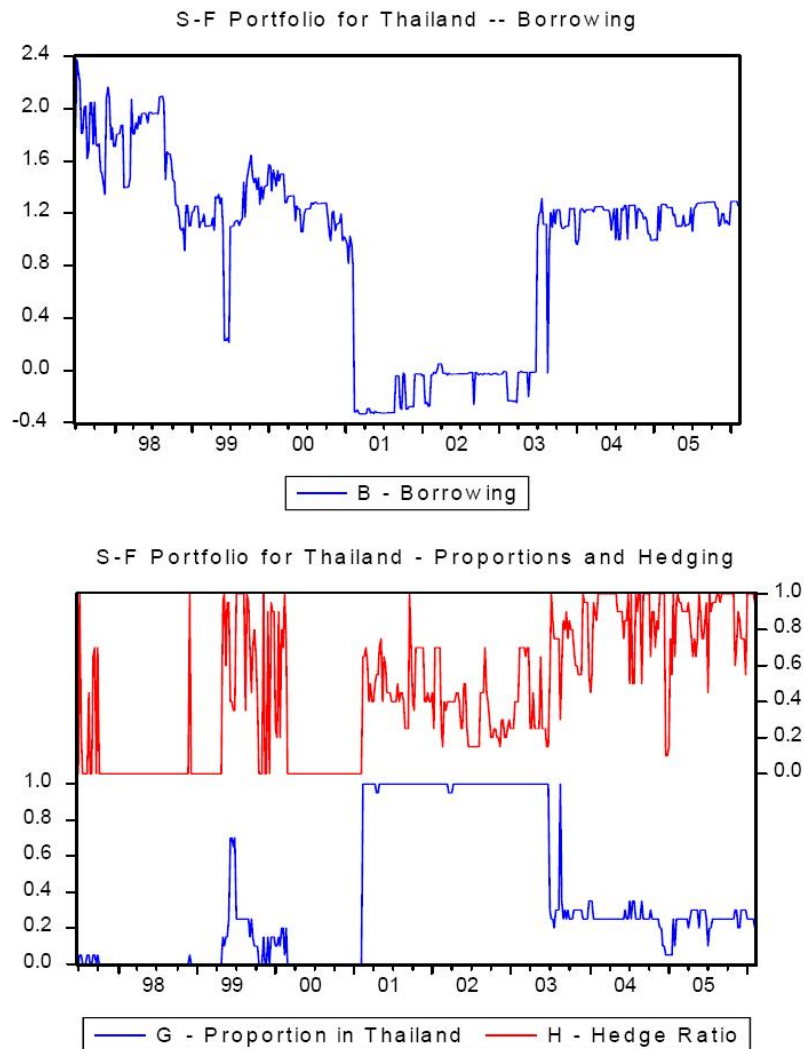


Fig. 29. S-F Portfolio for Thailand

Figure 30 compares the portfolio return in dollars of a 100% U.S. equity portfolio, a 100% Thai equity portfolio, as well as the dollar return on the risky portion of the safety-first portfolio, and the dollar return on the complete safety-first portfolio including risk-free investments. Note that the dollar return on (unhedged) Thai equity

is negative over our 1997 - 2006 period. The U.S. equity portfolio gains about 40% in value over this period, and the safety first portfolio gains almost 140% in value. Thus the safety-first portfolio involving Thai equity also outperforms a buy-and-hold strategy directed at U.S. equity, although again there is a substantial period where the decline in both nations equity values and the leveraging of portfolio values resulted in falling values of the safety-first portfolio and a period where the safety-first portfolio value would be below the value of a buy-and-hold portfolio of U.S. equity.

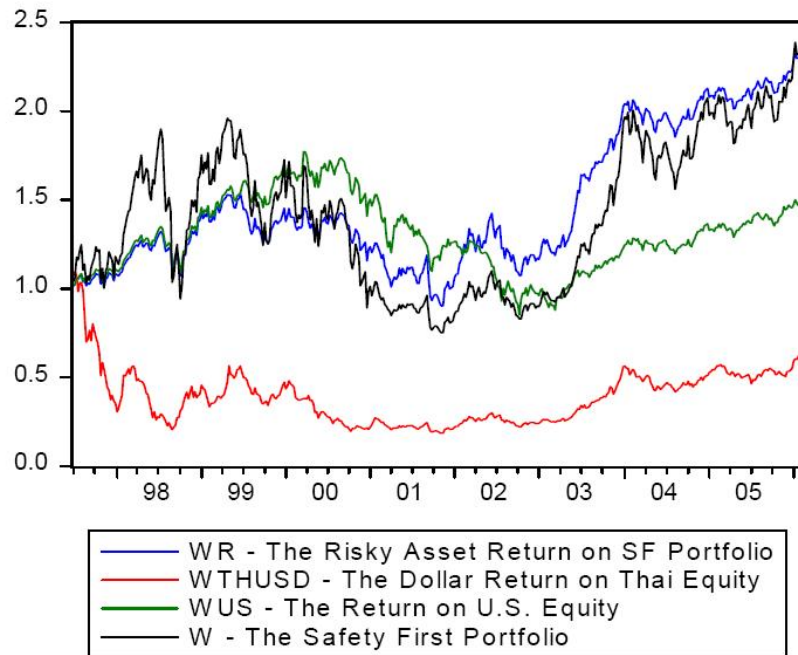


Fig. 30. Performance of S-F Portfolio Versus Country-Specific Index

H. Conclusion

We show that a dynamic safety-first portfolio selection procedure can be operationalized and would have provided limited downside risk for investors during the Asian

Crisis. In our portfolio simulations safety-first portfolios performed better than buy-and-hold portfolios of U.S. or foreign stocks. In fact, during the actual Asian Crisis period the safety-first portfolios outperformed the alternatives. There is a time during the U.S. stock market decline starting in 2000 when safety first portfolios underperformed buy-and-hold portfolios, largely because the safety-first portfolios were heavily leveraged. Our risk assumptions, especially setting the probability of a large decline equal to $1/520$ or once in ten years, led to our safety-first portfolios seeking in most periods to increase risk, i.e. to lever the portfolio. A buy-and-hold investor in the U.S. faces a much lower than $1/520$ chance of a 20% decline in one week in her portfolio value.

We also find the both hedging and portfolio proportions are important. Hedging was often large, but not always one, despite the fact that our model had no cost of hedging. Further, hedge ratios changed dynamically and sometimes dramatically, depending on market conditions.

CHAPTER V

CONCLUSION

In Chapter II, we analyze the performance of seven different VaR models in terms of portfolio performance. VaR is widely used as a measure of risk. Many methodologies have been developed to estimate VaR. We know that different estimators can vary substantially across VaR models. And these methodologies can be compared and criticized on the basis of economic theory, but also on the basis of observed performance. By using coverage probabilities, Bao et al (2006) study predictive accuracy of seven different estimators during the Asian crisis. We take a different approach to evaluating VaR estimators and look at their performance when used to guide the portfolio selection of financial practitioners. We look at the investors with asymmetric preferences towards portfolio returns and subjected to a downside risk constraint. We then demonstrate safety first portfolio selection guiding by seven VaR models, and investigate VaR estimator performance in terms of portfolio performance over the Asian crisis period. Our findings indicate the main disparity across methods for the investor is in the optimal amount of borrowing, and in terms of portfolio performance the VaR estimator with the best coverage probability is not necessarily the best estimator.

Chapter IV extends Chapter II by allowing the safety-first investor to hedge his foreign currency hedging. We find both the hedging choice and portfolio proportions are important to the investor. In sample period, most of the time safety-first portfolios performed better than buy-and-hold portfolios of U.S. or foreign stocks.

We investigate the dynamic relation between stock returns and trading volume of individual stocks in Chapter III. A new model has been constructed, in which stock price takes time to adjust to its equilibrium level. In our framework, the investors

submit limit orders, therefore, more than one transaction price exists in each trading period and the instantaneous demand may not equal to supply at each transaction price. Since our model cannot be solved analytically, a trading platform has been developed by Su (2007) to simulate the trading process. The simulation results suggest that during the price adjustment periods relatively low trading volume predicts a large absolute value in future price change. We apply a mixture normal approach to estimate the relationship between daily return and lags of volume for individual stock. We find relatively low past volume indicates a relatively large price movement in the future, which is consistent with the prediction of the model.

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APPENDIX A

DOWNSIDE RISK CONSTRAINTS AND CURRENCY HEDGING IN
INTERNATIONAL PORTFOLIOS: THE ASIAN CRISIS (CHAPTER IV)

The following graphs provide further information on the behavior of portfolios and country stock indices over time. Figure 31 and 32 provide information on Indonesia, Figure 33 and 34 on Korea, and Figure 35 and 36 on Thailand.

For Indonesia, the top panel in Figure 31 graphs the return on U.S. equity, the portfolio performance starting from mid-1997, all compared to the -20% safety-first constraint. Clearly a U.S. equity portfolio would never violate the safety-first constraint over this period. In a sense, it is too safe for a safety-first investor willing to accept a 1-in-520 chance of a -20% return.

The bottom panel of Figure 31 graphs the dollar return on an unhedged portfolio of Indonesian securities. There are many violations of the -20% constraint, especially in 1997 and 1998 but even as late as 1999.

The top panel of Figure 32 graphs the dollar return on the safety-first portfolio, as shown in Figure 20 of the text, but also indicates the number of violations of the safety-first constraint. As is apparent, the safety-first portfolio has a single violation of the safety-first constraint over the mid-1997 through 2005 period, nearly matching the 1-in-520 chance we specified for this portfolio choice. The safety-first portfolio achieves this by leverage, basically accepting more risk than a U.S.-only or Indonesia-only portfolio in exchange for a higher expected (and over here, actual) return.

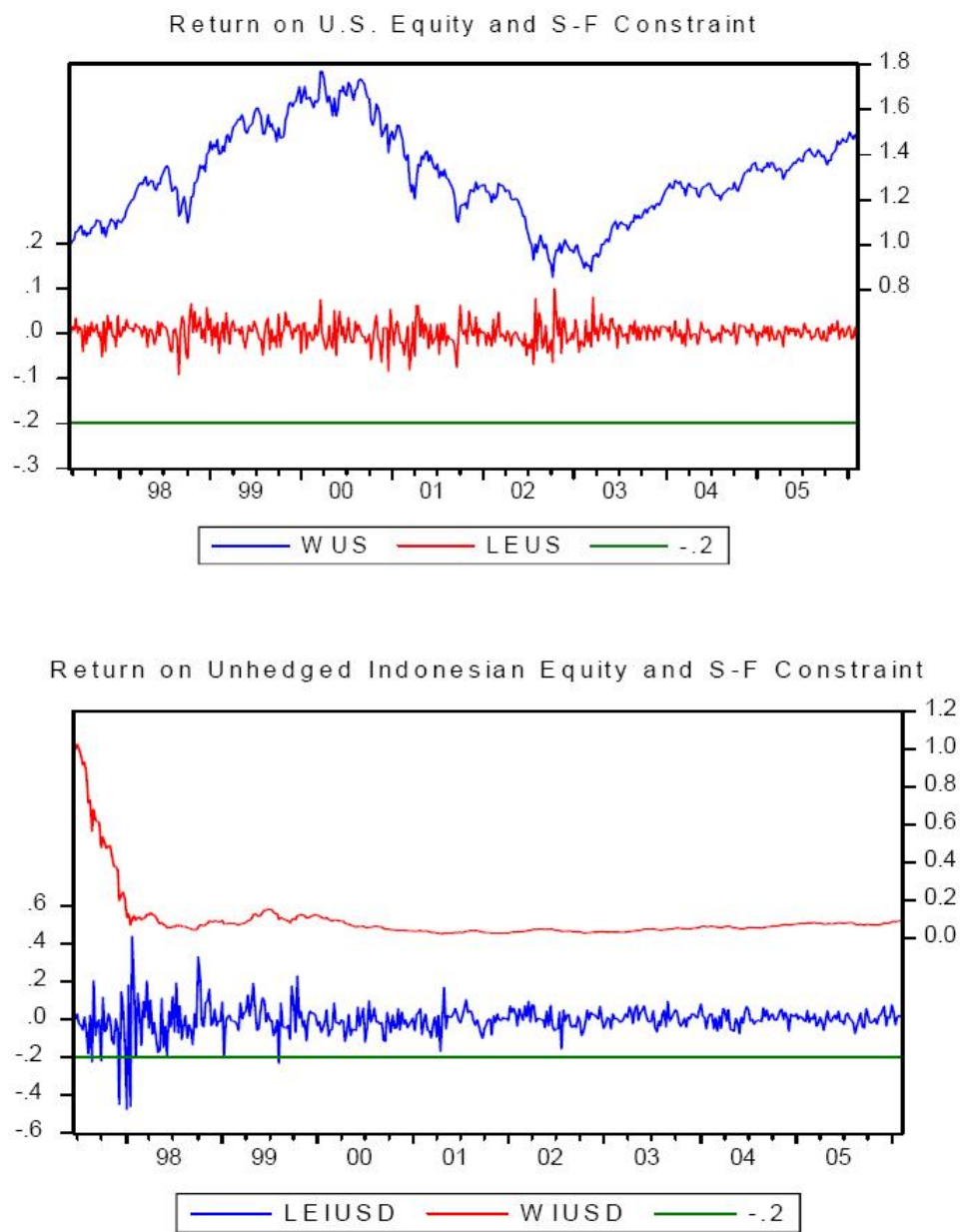


Fig. 31. Returns on U.S. and Indonesian Equity 1

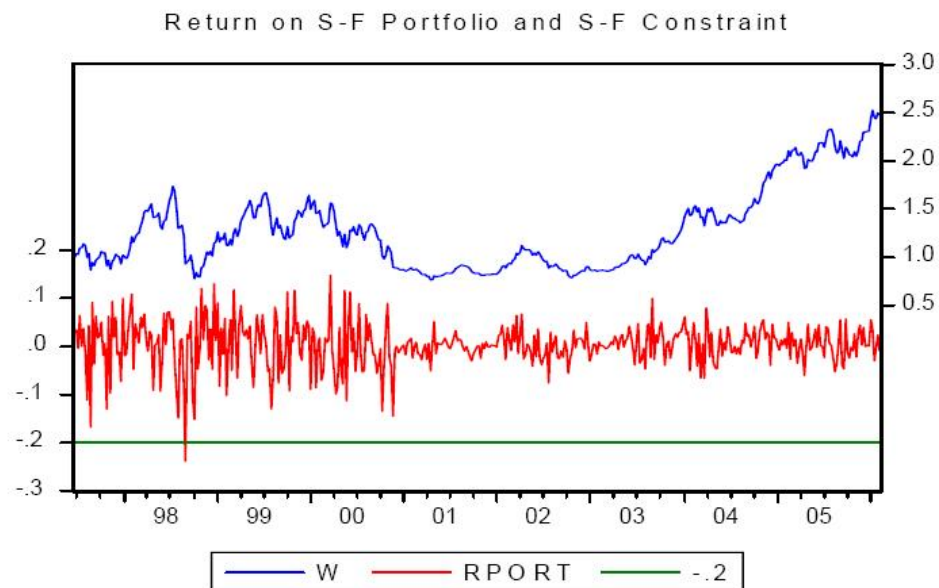


Fig. 32. Returns on U.S. and Indonesian Equity 2

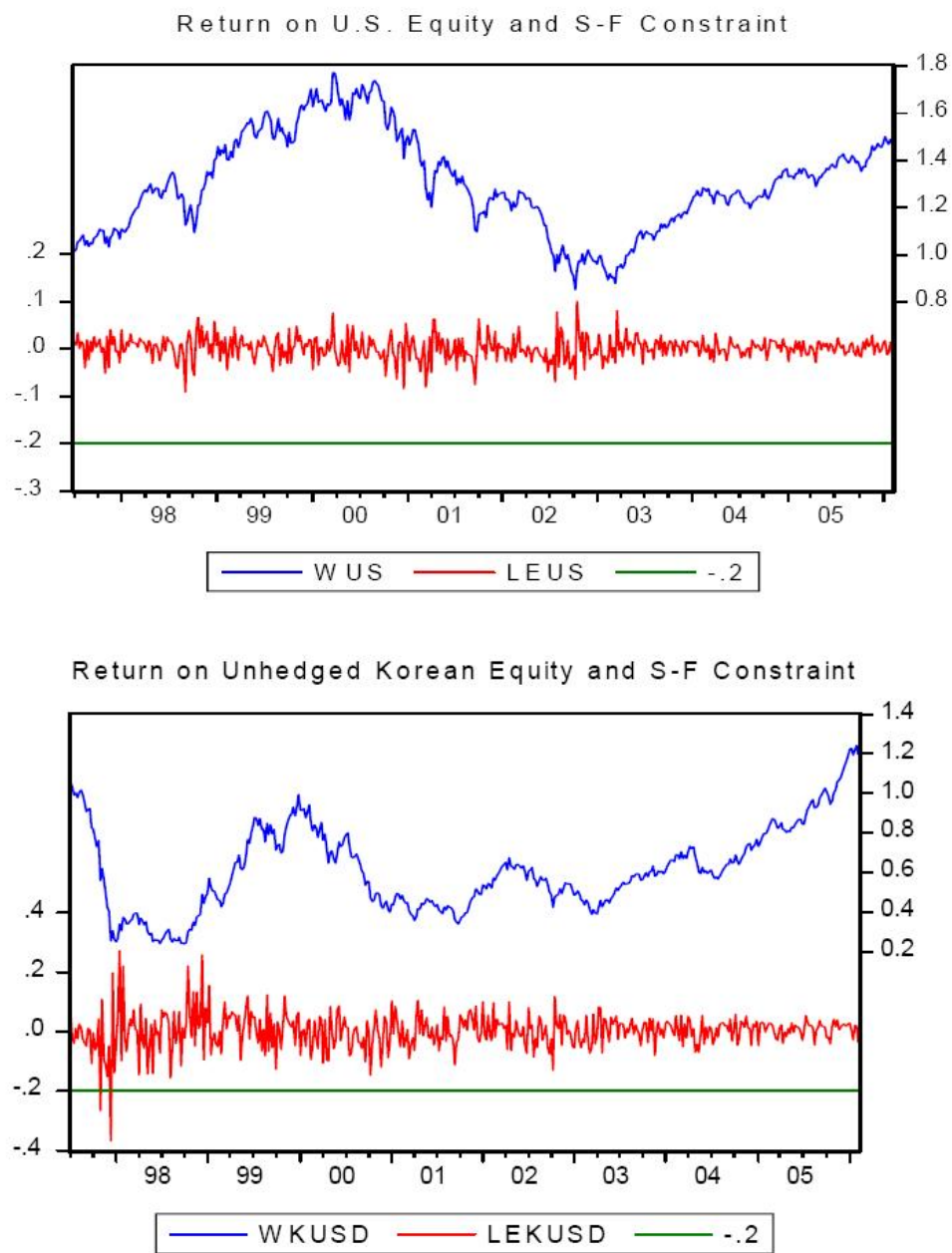


Fig. 33. Returns on U.S. and Korea Equity 1

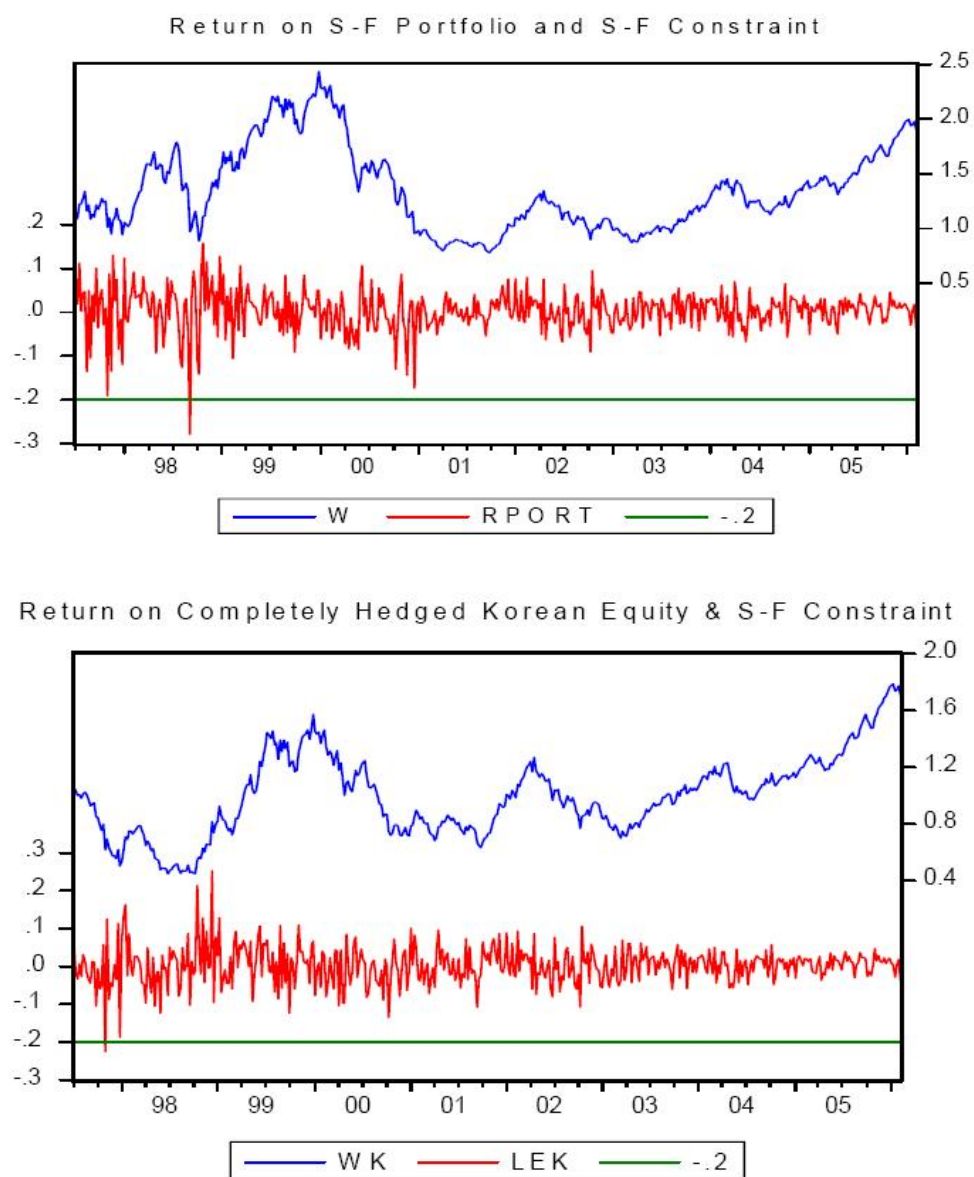


Fig. 34. Returns on U.S. and Korea Equity 2

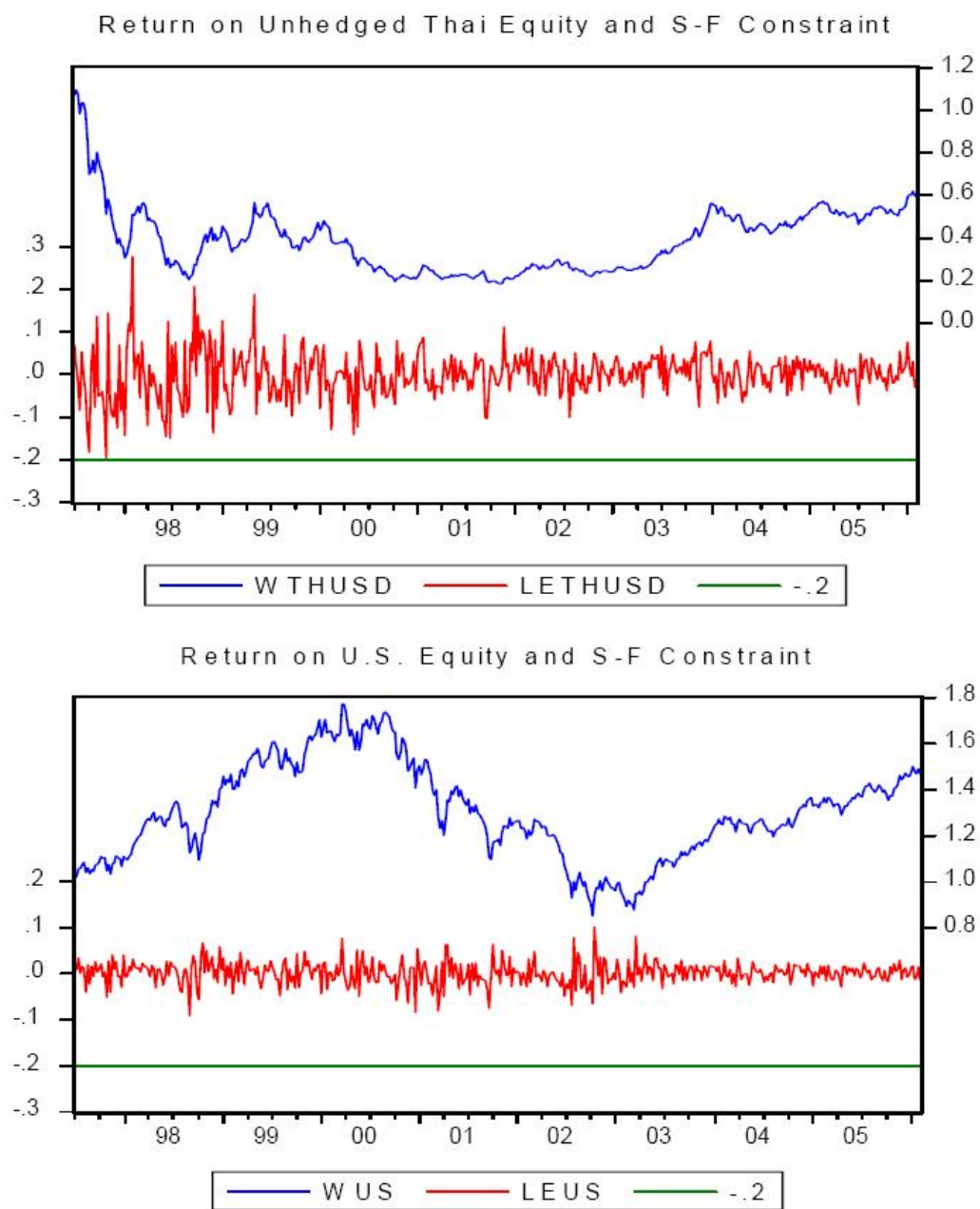


Fig. 35. Returns on U.S. and Thai Equity 1

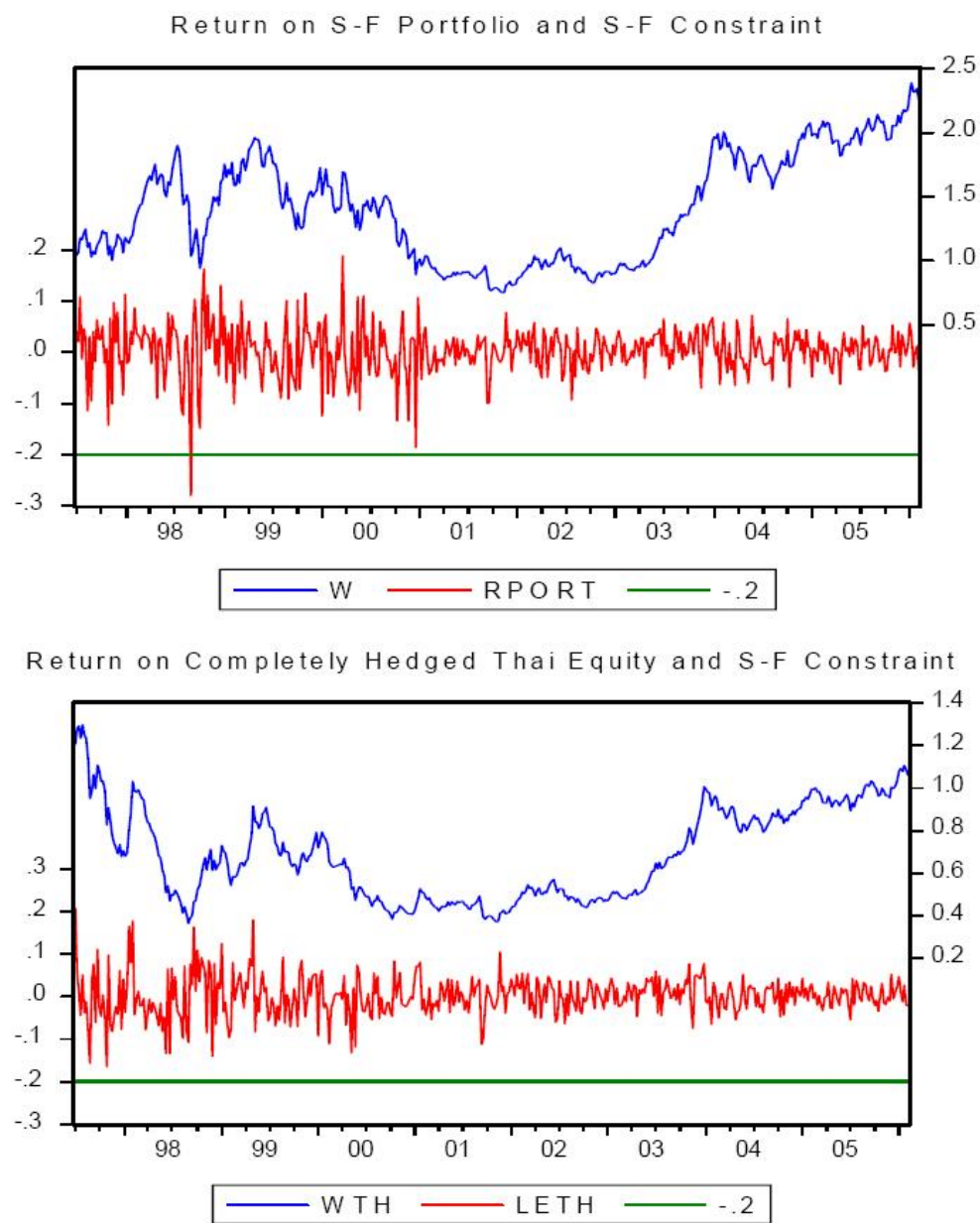


Fig. 36. Returns on U.S. and Thai Equity 2

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